Overview

- Least cost system operation
  - Economic dispatch
- Constrained optimization
  - Linear programming
  - Formulating and solving the Lagrangian
- Interpretation of economic dispatch results
  - Interpreting marginal cost and ‘System λ’

Recap: Example

- What is the economic dispatch for a two generator problem with:
  (The cost curve units are $/hr)

\[
C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2
\]
\[
C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2
\]
\[
P_{G1} + P_{G2} = P_L = 500 \text{ MW}
\]
Recap: Example

- Formulate the Lagrangian
- Take derivatives
- Solve
  \[ \frac{\partial L(P_G)}{\partial P_G} = 20 + 0.02P_G - \lambda = 0 \]
  \[ \frac{\partial L(P_G)}{\partial P_G} = 15 + 0.06P_G - \lambda = 0 \]
  \[ 500 - P_G - P_G = 0 \]

Economic Dispatch: Formulation

- For our problem, we find that
  - \( P_{G1} = 312.5MW \)
  - \( P_{G2} = 187.5MW \)
  - \( \lambda = $26.25/MWh \)
- Question: What happens (to the value of the objective function) if we dispatch the generators at output levels different from the constrained optimization solution?
  - (Note also: The values of the decision variables change with each value for \( P_L \))

Recap: Example

- Solve these three linear equations

\[
\begin{align*}
20 + 0.02P_{G1} - \lambda &= 0 \\
15 + 0.06P_{G2} - \lambda &= 0 \\
500 - P_{G1} - P_{G2} &= 0
\end{align*}
\]

\[
\begin{bmatrix}
P_{G1} \\
P_{G2} \\
\lambda
\end{bmatrix} =
\begin{bmatrix}
312.5MW \\
187.5MW \\
$26.25 / MWh
\end{bmatrix}
\]

Economic Dispatch: Interpretation

- Question: What happens (to the value of the objective function) if we dispatch the generators at output levels different from the constrained optimization solution?

\[ C_T = (1000 + 20P_{G1} + 0.01P_{G1}^2) + (400 + 15P_{G2} + 0.03P_{G2}^2) \]

\[ \frac{\partial x(P_{G1})}{\partial P_{G1}} = 20 + 0.02P_{G1} - \lambda = 0 \]
\[ \frac{\partial x(P_{G2})}{\partial P_{G2}} = 15 + 0.06P_{G2} - \lambda = 0 \]
Significance of Marginal Costs

• From total cost to marginal cost...
• The marginal cost is one of the most important quantities in operating a power system
  – the cost of producing the next increment of power (the next MW)
  – (Marginal cost = incremental cost)

Recall: Generator Cost Curve
Plots $/hr as a function of \( P_{\text{gen}} \) (MW) output

Recall: Marginal Cost Curve
Plots the $/MWh as a function of \( P_{\text{gen}} \) (MW) output

Marginal Cost Discussion
• With \( P_L \) unchanged, find the following:
  – MC equation for each generator
  – MC value at operating point (312.5MW, 187.5MW)
  – What is the total cost of serving this load?
  – What is the total cost if \( P_{G1} \) generates 325MW?
  – What is the MC of each generator at these new dispatch points? (325MW, 175MW)

\[
C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2
\]
\[
C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2
\]
Super-Important Results

• The traditional objective in operating a power system is to ___________________?
• The operating point found is the ____<in terms of cost>____ operating point.
• Operating anywhere else, thus ________________
• At the least cost operating point the incremental cost of all generating units ______________
• This incremental cost is the Lagrange multiplier, $\lambda$.
  – For power systems, this is the system-wide cost of generating electricity

Now Include Inequality Constraints

\[
\begin{align*}
\text{min } C_T &= \Sigma C_i(P_{Gi}) \\
\text{s.t. } &\Sigma(P_{Gi}) = P_L \\
& P_{Gi\min} \leq P_{Gi} \leq P_{Gi\max}
\end{align*}
\]

• For Homework: Iterate
• The field of operations research includes theory to solve as one unified problem.

ED With Inequality Constraints

• What is the economic dispatch for a two generator problem with

\[
\begin{align*}
C_1(P_{G1}) &= 1000 + 20P_{G1} + 0.01P_{G1}^2 \\
C_2(P_{G2}) &= 400 + 15P_{G2} + 0.03P_{G2}^2 \\
P_{G1} + P_{G2} &= P_L = 500\, MW \\
P_{G1} &\leq 300\, MW
\end{align*}
\]

ED With Inequality Constraints

• For our problem, we previously found that
  – $P_{G1} = 312.5\, MW$
  – $P_{G2} = 187.5\, MW$
  – $\lambda = 26.2/\text{MWh}$
• But now $P_{G1}$ output must be $\leq 300\, MW$, so what should we do?
ED With Inequality Constraints

• Interpret our results
  – What is system-lambda now, and what does it represent?
  – What inefficiencies have been introduced into our solution as a result of the binding generator limit?

ED With Inequality Constraints

• Note, for the HW
  – If we have three generators (or more), and you find that one of them must be set equal to a limiting value,
  – Then we have two generators remaining, and we cannot simply 'observe' the economic dispatch solution...
  – In this case, re-solve the Lagrangian problem with the remaining 2 generators, and the value $P_L$ adjusted as is appropriate

Problem Statement

• A power system has four generators with the following cost characteristics
  – $C_1 = 1000 + 15P_1 + 0.05P_1^2 \$/MWh
  – $C_2 = 1200 + 25P_2 + 0.12P_2^2 \$/MWh
  – $C_3 = 2060 + 20P_3 + 0.01P_3^2 \$/MWh
  – $C_4 = 2500 + 12P_4 + 0.03P_4^2 \$/MWh

• Typical demand, $P_L$, levels are:
  → 750MW, 1000MW, 1500MW, and 2500MW
Problem Formulation

• Write the equations you would use to find the least cost dispatch for this system
• How might you go about finding the least cost dispatch?
• What form will your results take?

Linear Algebra Solution Setup

<table>
<thead>
<tr>
<th>Matrix A:</th>
<th>Vector B:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 0 0 0 -1</td>
<td>-15</td>
</tr>
<tr>
<td>0 0.24 0 0 -1</td>
<td>-25</td>
</tr>
<tr>
<td>0 0 0.02 0 -1</td>
<td>-20</td>
</tr>
<tr>
<td>0 0 0 0.06 -1</td>
<td>-12</td>
</tr>
<tr>
<td>1 1 1 1 0</td>
<td>2500</td>
</tr>
</tbody>
</table>

• Solve in Matlab using \( x = (A)^{-1} \cdot (B) \)
• Where \( x = ? \)

Matlab Results

• For changing load levels:

<table>
<thead>
<tr>
<th>PL</th>
<th>PG1</th>
<th>PG2</th>
<th>PG3</th>
<th>PG4</th>
<th>Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>123</td>
<td>9</td>
<td>363</td>
<td>254</td>
<td>27</td>
</tr>
<tr>
<td>1000</td>
<td>154</td>
<td>22</td>
<td>518</td>
<td>306</td>
<td>30</td>
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<tr>
<td>1500</td>
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<tr>
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<td>1137</td>
<td>512</td>
<td>43</td>
</tr>
<tr>
<td>2500</td>
<td>339</td>
<td>100</td>
<td>1446</td>
<td>615</td>
<td>49</td>
</tr>
</tbody>
</table>

(the units are ___?)
Matlab Results → Interpret …

The solution is:
- \( P_L = 2500 \text{ MW} \)
- \( P_{G1} = 339 \text{ MW} \)
- \( P_{G2} = 100 \text{ MW} \)
- \( P_{G3} = 1446 \text{ MW} \)
- \( P_{G4} = 615 \text{ MW} \)
- \( \lambda = 49/\text{MWh} \)

Results Plotted

Visualizing Equal Marginal Cost; Quadratic Cost Curves Graphed

Visualizing Equal Marginal Cost
Including Inequality Constraints

- Include $P_{\text{min}}$ and $P_{\text{max}}$ values
  - Each generator must generate more than 50MW
  - $P_1$ and $P_2$ must generate less than 500 MW
  - $P_3$ and $P_4$ must generate less than 1400 MW
- How would you formulate the problem with these inequality constraints?

Summary

- Develop the mathematical origin for generator costs
  - Define heat rate
- Formulate the economic dispatch problem conceptually
- Develop mathematical formulation for solving the economic dispatch problem
- Interpret the results, including the Lagrangian multiplier and marginal cost