A Dynamic Model of Romeo and Juliet's Relationship

(adapted from MIT's signals course)

Below is a model for the waning and waxing of Romeo and Juliet's love for each other.

The continuous time model can be described with the following two equations

$$\dot{R}(t) = \alpha J(t) + \gamma R(t)$$
$$\dot{J}(t) = \beta R(t) + \delta J(t)$$

Or in matrix (also "state space") format, these equations are

$$\begin{bmatrix} \dot{R}(t) \\ \dot{J}(t) \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

- R(J) is the love of Romeo for Juliet. It is positive when the love is strong and negative when Romeo doubts his feelings
- α is the degree that Romeo responds to Juliet's feelings.
- γ is the degree that Romeo responds to his own feelings. It is negative when he is cautious and positive when he wants to burn with his love.
- J(R) is the love of Juliet for Romeo, and is positive and negative analogous to Romeo's feelings
- β is the degree that Juliet responds to Romeo's feelings
- δ is the degree that Juliet responds to her own feelings. It is negative when she is cautious and positive when she wants to burn with her love.

Anticipated Results

- If R(t) or J(t) increase without bound, then Romeo (or Juliet) will burn.
- If either variable R(t) or J(t) decrease to zero, then their love has been crushed.
- A good outcome, of a stable, loving relationship, would be with R(t) = J(t) > 0

Sample System Matrices

1. Initial conditions: x(0) = [1 1]

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \dot{\mathbf{x}} = \mathbf{Ax}$$

2. Initial conditions: x(0) = [0.1 10]

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

3. Initial conditions: x(0) = [0 15]

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

4. Initial conditions: x(0) = [10 10]

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

5. Initial conditions: x(0) = [0.1 0.1]

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

6. Initial conditions: x(0) = [10 10]

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

7. Initial conditions: x(0) = [1 0.1]

$$A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$