EGR 301 Homework 7

Chapters 4 and 5: Linear Systems Analysis

Note: For all problems involving computer modeling, hand in your Matlab script (*i.e.,* a '.m' file; well-commented), along with the relevant results.

Problem 1: Chapter 5, #6, page 179 – you may select reasonable values for the coefficients and solve this using Matlab, as long as you detail (or include in a script file) every step necessary to diagonalize the matrix.

Problem 2: Chapter 5, #8, page 180 – as above, you may select reasonable parameter values and use Matlab to solve.

Problem 3: The Continuing Romeo and Juliet Relationship Model

The objective of this problem is to continue the development of a model for the waning and waxing of Romeo and Juliet's love for each other. The model will be simulated and suggestions (in terms of system inputs) will be made to help stabilize the relationship.

The CT model can be described with the following two equations

$$\dot{R}(t) = \alpha J(t) + \gamma R(t) + aU_1(t)$$
$$\dot{J}(t) = \beta R(t) + \delta J(t) + bU_2(t)$$

Or in state space format, these equations are

$$\begin{bmatrix} \dot{R}(t) \\ \dot{J}(t) \end{bmatrix} = \begin{bmatrix} \gamma & \alpha \\ \beta & \delta \end{bmatrix} \begin{bmatrix} R(t) \\ J(t) \end{bmatrix} + \begin{bmatrix} a \cdot K_1 \\ b \cdot K_2 \end{bmatrix} u(t)$$

- R(J) is the love of Romeo for Juliet. It is positive when the love is strong and negative when Romeo doubts his feelings
- α is the degree that Romeo responds to Juliet's feelings.
- γ is the degree that Romeo responds to his own feelings. It is negative when he is cautious and positive when he wants to burn with his love.
- a is a binary coefficient indicating whether or not Romeo is listening to the advice of others.
- J(R) is the love of Juliet for Romeo, and is positive and negative analogous to Romeo's feelings
- β is the degree that Juliet responds to Romeo's feelings
- δ is the degree that Juliet responds to her own feelings. It is negative when she is cautious and positive when she wants to burn with her love.

- b is a binary coefficient indicating whether or not Juliet is listening to the advise of others.

Anticipated Results

- If R(t) or J(t) increase without bound, then Romeo (or Juliet) will burn.
- If either variable R(t) or J(t) decrease to zero, then their love has been crushed.
- A good outcome, of a stable, loving relationship, would be with R(t) = J(t) = 1

TASKS to COMPLETE:

Initial tasks

1. Assume that the initial condition for Romeo is love a first sight (which is to say that R(0) = 1) and that Juliet has not yet seen Romeo (so J(0) = 0). Identify and find values for the remaining initial conditions required to find the total solution to this system (think back to EGR 220 and the initial conditions required to solve second order circuit problems).
2. As was done in class, derive the second order, differential (a.k.a. input-output) system equations
   a) In terms of R(t) alone
   b) In terms of J(t) alone

Zero-Input Response (Natural Response)

3. Assume that $\alpha = 1$ and that $\beta = -1$. Also assume that so far neither Romeo nor Juliet at getting outside advice. Finally, assume that $\delta = \gamma = 0$. (this is system example #4 from the initial Romeo and Juliet handout)
   a) Plot R(t) and J(t) on the same graph
   b) Perform an eigenanalysis of the system, including a brief explanation of the system behavior.
4. Explain the behavior of the systems represented by system examples #5, 6 and 7 (from the initial R&J handout) in terms of eigenvalues and eigenvectors (as done in class on 4/5 for examples #1, #2 and #3).

Total Response with System Input ('open loop input or open loop control')

5. Make the same assumptions as in step 3, except that a = 1 now, and let the input be $K_1u(t)$ (*i.e.,* $K_2 = b = 0$). Plot R(t) and J(t) on the same plot, trying different values for K_1. Identify the plot you like best (*i.e.,* the K_1 value you like best), and briefly explain why you have selected it.
6. Now let b = 1 as well as a = 1, with both K_1 and K_2 non-zero. Plot R(t) and J(t) on the same plot, trying different values for K_1 and K_2. Identify the plot you like best (*i.e.,* the K values you like best), and briefly explain.
7. Comment on which version of the system (choosing *any* of the system versions we have analyzed) you think is best for Romeo and Juliet, and briefly explain why.