EGR 301 Homework 5

Differential Equations

Note: For all problems involving computer modeling, hand in your Matlab script (*i.e.,* a '.m' file; well-commented), a listing of the numbers in the sequence determined by your script and a labeled plot, as is relevant to the problem.

Problem 1: A car weighing 3000 lb is put in motion on a level highway and then allowed to coast to a stop. Its speed is measured at successive increments of time, as recorded in the following table:

Time (sec)	Speed (ft/sec) approx.
0	15.2
10	10.1
20	6.6
30	4.5
40	2.1
50	1.25

a) Using a mass-damper model for this system, draw the system diagram and set up the differential equation for the velocity v of the vehicle.
b) Evaluate m and then estimate b using a graph of the data given in the table above. (Hint: Use the slope dy/dt and v itself at any time t.)

Problem 2: Using the harmonic motion Matlab scripts from class, characterize the effect on the system behavior of changing various values:

a) Changing (increasing and decreasing) the parameters m and k. In particular, find the relative values of m and k that lead to under-, over- and critically damped system behavior.
b) Changing the initial conditions, and
c) Changing the frequency of the forcing function, to less than, greater than, and equal to the natural or resonant frequency of the system (note that you will need to select reasonable values for m and k for this question).

Problem 3: Write your own Matlab program using the ODE45 solver

a) You are free to choose any second order equation (system) to model. You could, for example, model a mass-spring-damper system, or any similar mechanical system or circuit we have discussed in this class or anything that you have seen in other courses or think of on your own. (You cannot use the same system as in problem 2, which involves only a mass and spring, but no damper.)
b) Provide two or more plots showing the system behavior under different conditions (*e.g.,* with different parameter values, or different initial conditions…). Comment upon, or briefly discuss, your plots (*e.g.,* what did you do and why is it interesting?)

Problem 4: Chapter 3, problem 10 (page 87)

Problem 5: Matrix transformations

a) Example 1 on page 74 of the text discusses "book rotations." This example presents two rotation matrices **A** and **B**.
b) Confirm that **AB** ≠ **BA** by multiplying the matrices by hand (which is to say, to not use Matlab, except for verifying your results if desired).
c) Deduce and demonstrate (via a Matlab script and appropriate plot(s)) what the effect (type of rotation) is produced by the matrix **AB.**
    a. This is to say, if vector **u** is multiplied by **AB**, then the vector **ABu** has what orientation or rotation with respect to the original vector **u**?
d) Deduce and demonstrate (via a Matlab script and appropriate plot(s)) what the effect (type of rotation) is produced by the matrix **BA.**
    a. *Do these two rotations for at least TWO DIFFERENT initial conditions* (*e.g.,* you can use v1 = [1 1 1]' for the vector that is being rotated, and at least one other linearly independent vector to be rotated. A good option would be to make one or more of the elements in v1 negative.
e) Briefly discuss (one or two sentences) the effect of the rotations, A, B, AB and BA, and why they appear to behave differently with different initial vectors.

Problem 6: Chapter 3, problem 12 (page 87). Note, for this problem you may use Matlab to find P^{-1} for you **IF** you also write out the basic procedure (the steps) for finding the matrix inverse. This is to say you do not need to do all the algebra out by hand as long as you convince me that you know how to find the inverse without a computer if you needed to do so.