EGR 301 Homework 4

Difference Equations & Familiar Dynamic Systems

<u>Note</u>: For all problems involving computer modeling, hand in your Matlab script (*i.e.,* a '.m' file; well-commented), a listing of the numbers in the sequence determined by your script and a labeled plot, as is relevant to the problem.

<u>Problem 1:</u> Text, chapter 2 # 3 (page 47)

a) Do the problem as stated in the text – finding y(k) for k = 3, 4, 5 – by hand (*i.e.,* showing the determination of y(3), y(4) and y(5) on paper, without the use of the computer)
b) Write a Matlab script to find and plot the first 9 terms of the sequence, using the difference equation from the problem statement.

<u>Problem 2:</u> Text, chapter 2 # 5

a) Do the problem as stated in the text. (Note that the equation can be readily deduced by observation and some obvious guesswork. Though there may be an elegant approach to the solution, observation and guesswork evolve into 'intuition' and 'experience,' and are perfectly valid approaches in this type of problem.)
b) Write a Matlab script to find and plot the first 10 terms of the sequence, using the difference equation from the problem statement.

<u>Problem 3:</u> Text, chapter 2 # 10, *part (a) only*

<u>Problem 4:</u> Text, chapter 2 # 17, *parts (d) and (e) only*

<u>Problem 5:</u> (adapted from linear algebra notes at Duke University)

A simplified model of a national economy has the form

$$\mathbf{y(k+2) - a(1 + b)\, y(k+1) + ab\, y(k) = 1}$$

where **y(k)** is the total national income during year **k**, **a** is the marginal propensity to consume (a constant < **1**), and **b** is a constant of adjustment, which describes how the rate of private investment is affected by changes in consumer spending.

To use this equation, we need to make two decisions/observations:

(1) Which year will be year '0,' our starting point? Note that the difference equation defines **y(k+2)** in terms of **y(k+1)** and **y(k)**. In order to generate a sequence from this equation, we must know the initial conditions **y(0)** and **y(1)**.
(2) What are reasonable sizes for numbers for **y**, a national income? We can observe that the left-hand side is a weighted sum of incomes, and it must add up to **1**. Therefore

we can assume that the monetary unit has been scaled so that the order of magnitude of an annual income is **1**. Assume that one unit represents one trillion dollars.

<u>Tasks</u>: In the tasks below, you will be modifying the initial conditions and the system parameters, and observing the affect of these changes on the system behavior. Note that in the following weeks we will be learning systematic methods to quantify and understand how various aspects of any dynamic system affect the overall system behavior.

a) Start with **a = 0.9**, **b = 0.5**, **y(0) = 1**, and **y(1) = 1.1**. Calculate the national income for **30** years, and plot the sequence as a function of **k**. ***Briefly*** discuss or explain what you observe.
b) Experiment with starting amounts. What happens if the income drops to 0 in year 1? What happens if it is 0 in year 0, but positive in year 1? Try other combinations as well. ***Briefly*** discuss the influence of these starting values have on the final, steady-state of the economy.
c) Set the starting amounts to give whichever plot you found most interesting in step (b). Now experiment with the adjustment constant **b**, both raising it and lowering it (but keep it positive). Discuss the influence of **b** on the final state of the economy.
d) Finally, return **b** to its first value of **0.5**, and experiment with '**a**,' trying a range of values between **0** and **1**. How much influence does '**a'** have on the final state of the economy?

<u>Problem 6: System Examples – Observing your world</u>

- Your answers to this problem will be discussed in class on March 1
- *Assignment:* Observe the world around you and identify your own system example. Using this system example:
  - Identify the system inputs, outputs, parameters
  - Identify the system components, any subsystems, and/or the internal variables
    - Think about how the components are coupled (strongly, weakly, not at all…)
  - Discuss briefly how an analysis of this system could be used
    - What groups of people would be interested in understanding the system evolution, performance, behavior?
    - What groups of people would be interested in understanding the system outputs, and in predictions of future outputs and behavior?
  - Draw a block diagram illustrating your understanding of the system
    - Identify the variables, coupling between variables/subsystems and feedback (connections of output from a subsystem back to the input of the same or a different subsystem)
    - Begin the diagram with the input (on the left) and have it end with the output coming out (on the right)
  - Hand in your own system description, but feel free to talk with each other to get ideas about your system
- Examples of systems, to get you started thinking, are introduced on page 1 of the text, as well as throughout the book.