

Second Order Circuits III

EGR 220, Sections 8.5 – 8.7

October 21, 2008

Terminology Review

➤ Define and describe how we find:

◦ Characteristic equation $s^2 + \frac{R}{L}s + \frac{1}{LC}$ (series)

◦ Natural frequencies $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

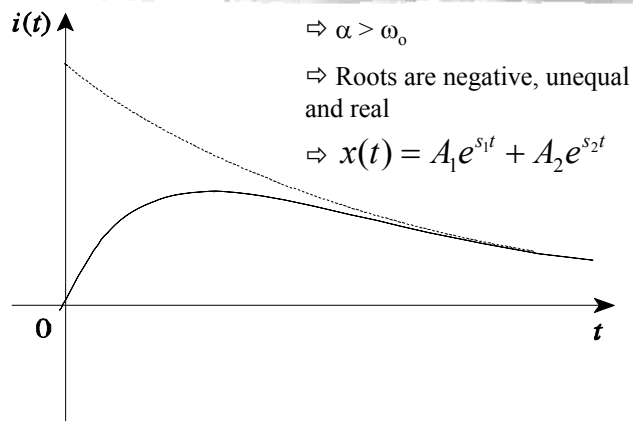
◦ Resonant frequency $\omega_0 \equiv \frac{1}{\sqrt{LC}}$

◦ Damping factor $\alpha \equiv \frac{R}{2L}$ (series) $\equiv \frac{1}{2RC}$ (parallel)

◦ Damping frequency – for underdamped systems

2

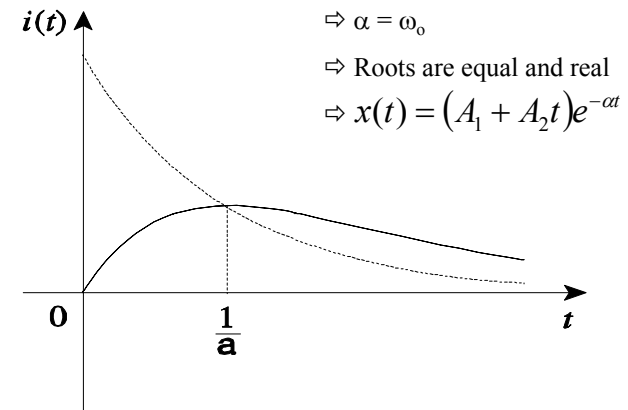
Overdamped Response



(a)

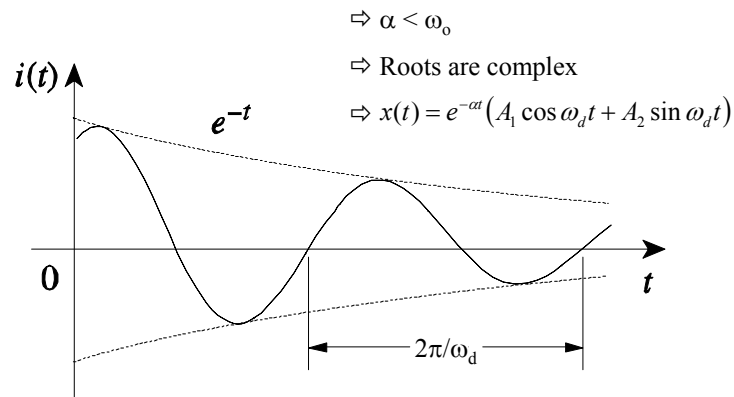
3

Critically Damped Response



4

Underdamped Response



5

Concept Review (review Q 8.5)

- In a series RLC circuit, setting $R = 0$ will produce
- an overdamped response
 - a critically damped response
 - an underdamped response
 - an undamped response
 - none of the above

6

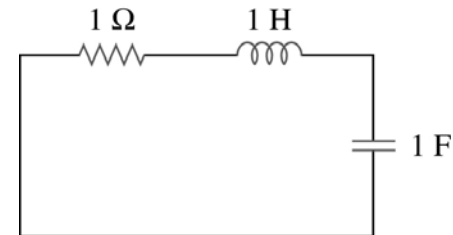
Concept Review

- RLC properties, pages 323-324
- What determines the *rate* of damping?
 - Adjusting *what* shifts the response between over-, critically- and underdamped?

7

Concept Review (review Q 8.7)

- The response of the circuit below will be
- overdamped
 - underdamped
 - critically damped
 - none of the above



8

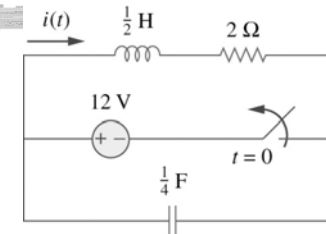
Solution Method

- List the steps for find the natural response of a second order circuit
 - a)
 - b)
 - c)
 - d)
 - e)
 - f)
- How are all $v(t)$ and $i(t)$ signals related in a single circuit?

9

Natural Response Example

- Find $i(t)$ for $t > 0$ (Problem 8.20)



10

Second Order Circuit Analysis

... with sources

11

Series RLC Circuit Behavior

- The general approach is to apply KVL and solve the resultant equation...

$$v_R + v_L + v_C = V_s$$

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int_{-\infty}^t idt = V_s$$

12

Series RLC Circuit Behavior

➤ ... apply KVL and substitute...

Using $i = C \frac{dv}{dt}$... this becomes

$$\frac{d^2v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{1}{LC} v = \frac{V_s}{LC}$$

13

Recap: Solution Format

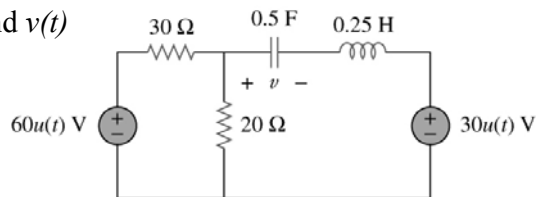
$$x(t) = X_f + X_n$$

$$x(t) = X_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

14

Step Response of Parallel RLC Circuit (problem 8.39)

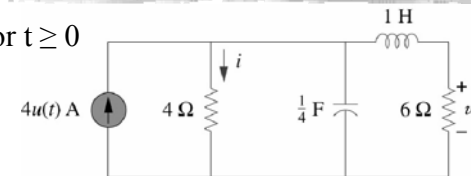
➤ How do we find $v(t)$ for $t > 0$?



15

General Second Order Circuit

➤ Find $i(t)$ and $v(t)$ for $t \geq 0$



16