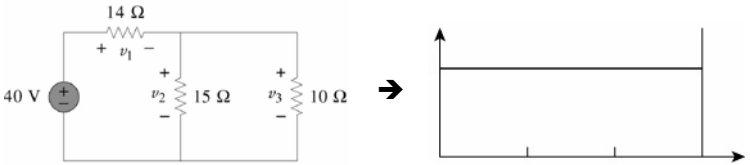
Second Order Circuits I: Initial Conditions

EGR 220, Chapter 8.1 – 8.2

October 9, 2005

Concept Map 3

Input, Sources → System → Output, Circuit Behavior



➢ DC, static circuits: Chapters 1 through 4

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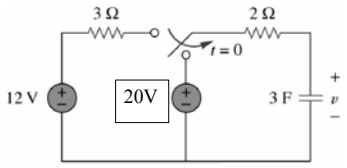
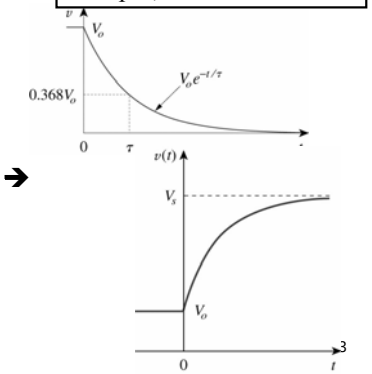


$$x_n(t) = X_0 e^{-t/\tau}$$
$$x_f(t) = X_S\left(1 - e^{-t/\tau}\right)$$
$$x_1(t) = X_o e^{-t/\tau} + X_s\left(1 - e^{-t/\tau}\right)$$
$$x_2(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau} = x_1(t)$$



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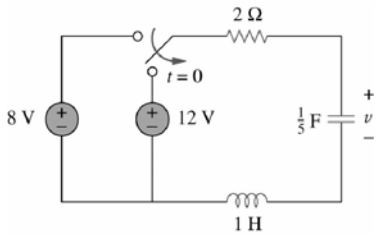
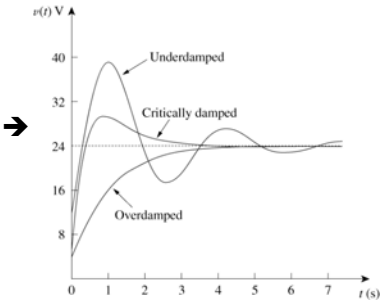




Overview

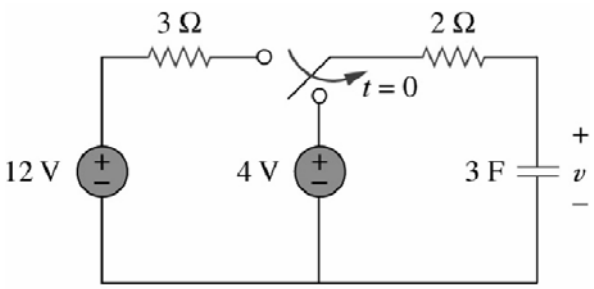
- Review of first order circuit solutions
- * Initial values for second order circuits *
- Circuit solutions
  - Source-free response – Today
  - Complete response – Next Thursday
- Damping of circuit response – next class

Recap: Step Response of RC & RL Circuits

- The step response is the complete response
- The complete (total) response is the sum of
  - The natural + the forced response
  - The transient + the steady state response
- Circuits may have a variety of inputs
  - Step input
  - (Sinusoidal input – next chapters)
  - (Ramp input; impulse (delta) input – EGR 320)

Recap: Step Response of an RC Circuit

- Find *v(t)* (prob. 7.40)



Recap: Step Response of an RC Circuit

- Three solution methods
  - Use *superposition* of natural response and forced response
  - Use text equation, applicable for the step response
  - (Use *transient* and *steady-state* response)
- Be comfortable with what every term in each solution method represents

Second Order Circuit Analysis

- "Second order circuits" means what?
  - Second order differential equations are required to describe the behavior of RLC circuits
  - Contain _____ storage elements?
- What do we want to discover through analyzing the circuit?
- What time periods are of interest?
- What initial information do we need?
- How do we proceed with the analysis?

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Second Order Circuit Analysis

- How we begin to find initial conditions
  - Focus on the variables that cannot change instantaneously, which are...
  - Capacitor (voltage/current)_____
  - Inductor (voltage/current)_____
- For a second order circuit, we need values for two initial conditions (two constants of integration)
  - (We also find the final values at t = ∞)

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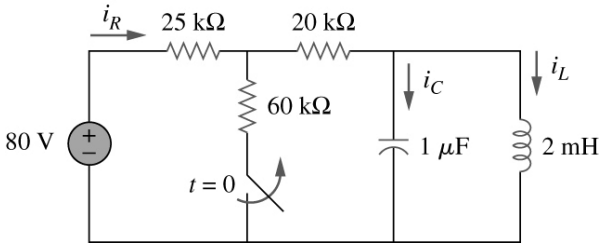
Initial Values (problem 8.2)

- Find: $i_R(0^+)$, $i_L(0^+)$ and $i_C(0^+)$,
- and: $di_R(0^+)/dt$, $di_L(0^+)/dt$ and $di_C(0^+)/dt$,
- and: $i_R(\infty)$, $i_L(\infty)$ and $i_C(\infty)$



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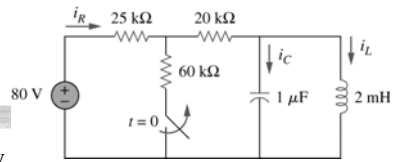
Initial Values 1

1. Write down everything we know about the circuit



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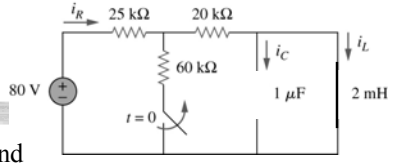
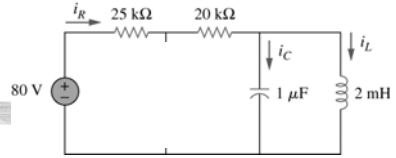
Initial Values 1



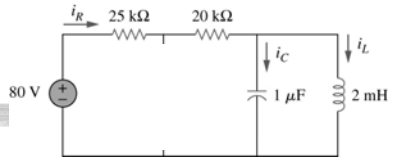
2. Begin solving: For time t = 0^- find
   - ⇨ $i_L(0^-)$ and
   - ⇨ $v_c(0^-)$ (Which also tell us what?)

Initial Values 1



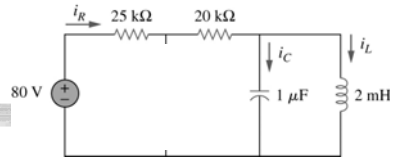
3. For time t = 0^+, find
   - ⇨ $i_R(0^+)$ (use KVL) and
   - ⇨ $i_c(0^+)$ (use KCL)

Initial Values 1



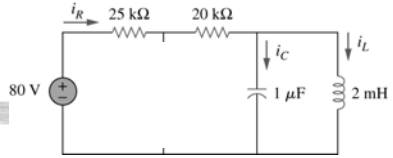
4. Now find the first derivatives:
   What do we know about $v_L(t)$ in general, and what can we deduce about $v_L(0^+)$?
   Use this to find $di_L(0^+)/dt$

Initial Values 1



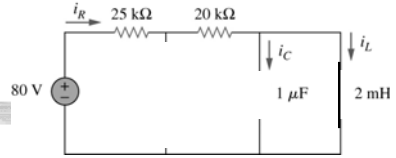
5. Finding $di_R(0^+)/dt$
   - ⇨ Use 'd(KVL)/dt' to find an expression for $di_R(0^+)/dt$
   - ⇨ Use $i_c = C(dv_c/dt)$ to solve for $di_R(0^+)/dt$

Initial Values 1



6. Finally, find $di_c(0^+)/dt$
   - ⇨ Use 'd(KCL)/dt' to find an expression for $di_c(0^+)/dt$

Initial Values 1



7. Find the final values, at $t = \infty$
   - ⇨ $i_L(\infty)$ =
   - ⇨ $i_R(\infty)$ =
   - ⇨ $i_C(\infty)$ =

Next Class: Circuit Response & Damping

- What are the three categories of damped response?
- What is being damped?
- What is doing the damping?
- What affects the type of damping?

Summary

- Second order circuits
  - Solving second order differential equations
  - Understanding energy storage and oscillatory circuit response
- Damping
  - Over-, critically and under-damped response