Chapter 8: Initial & Final Values Practice Problem

We are ultimately interested in finding different functions for $v(t)$ or $i(t)$ throughout the circuit for $t > 0$.

We are only interested in the circuit for time $t > 0$ (since that is given for this problem as the switching time).

We only pay attention to $t < 0$, exclusively for $V_c(t)$ and $i_c(t)$, only because we know these values continue over across the moment of time $t = 0$.

We actually want $V_c(0^+)$ and $i_c(0^+)$. But it is easier to find $V_c(0^-)$ and $i_c(0^-)$ since the circuit is in steady-state up to $t = 0$. And using $V_c(0^+) = V_c(0^-)$ and $i_c(0^+) = i_c(0^-)$, we find the easy-to-find values at $0^-$ and then have our first two pieces of information for time $t = 0^+$, i.e., $t > 0$.

After finding $V_c(0^-)$ and $i_c(0^-)$ we pay attention only to the form of the circuit for $t > 0$.

For this problem, practice finding lots of initial conditions $i_c(0^+), i_c(0^-), i_R(0^+), V_c(0^+)$

\[
\begin{align*}
\frac{di_c}{dt}, \frac{di_c}{dt}, \frac{di_R}{dt}, \frac{dV_c}{dt} \quad \text{all at } t = 0^+ \\
\mu_R(\alpha), \mu_c(\alpha), i_c(\alpha)
\end{align*}
\]
To start, find initial conditions

To repeat, what we actually need are the values for $t=0^+$
which is the initial moment of time for time $t>0$

But the way to find values for $i_c(0^+)$ and $v_c(0^+)$ is

- to use the continuity relationships of $v_c(0^+)=v_c(0)$ and $i_c(0^+)=i_c(0^-)$

- So we calculate $v_c(0^-)$ and $i_c(0^-)$ and then say we’d have now found what we want, i.e., $v_c(0^+)$, $i_c(0^+)$

For $t<0$ assume capacitor is an open circuit and the inductor is a short circuit (the electric and magnetic fields are fully charged)

Note that $L+C$ are in parallel so $v_c=V_c$

Since the inductor acts as a short circuit now, $V_c=0$

Therefore $V_c=0 \Rightarrow v_c(0^-)=0 \Rightarrow v_c(0^+)=0V$

The current through the inductor is the same as the current through the 20kΩ resistor

Using current divider to find $i_{20k}$ we first need the total current from the source, $I_s$

\[
i_{20k} = I_s \left( \frac{60}{20+60} \right)
\]

\[
I_s = \frac{V_s}{R_{eq}} = \frac{80}{25k + (60/20)k} = \frac{80}{(25+15)k} = 2 mA
\]

So $I_{20k} = 2m \left( \frac{60}{80} \right) = 1.5 mA = i_c(0^-)$ \quad $i_c(0^+) = 1.5 mA$
continuing with $i_c(0^+)$ and $i_r(0^+)$ we now must
redraw the circuit for $t > 0$!!

$t > 0$

for $i_r(0^+)$ use KVL

\[-80 + i_r(45k) + V_c = 0\]

for this time moment, $t = 0^+$, we know $V_c(0^+) = V_c(0) = 0V$
so $80 = i_r(45k) + 0 \Rightarrow i_r(0^+) = 1.78 mA$
The initial condition for $i_r(t)$

Find $i_c(0^+)$ using KCL

\[i_r = i_c + i_L \Rightarrow \text{we know} \cdot i_c(0^-) = 1.5 mA = i_L(0^+)\]

\[\cdot i_r(0^+) = 1.78 mA \text{ (found above)}\]

Therefore $i_c(0^+) = i_r(0^+) - i_L(0^+) = 0.28 mA$

Now to the first derivative initial conditions. Thinking ahead, we ultimately need to find values for the constants in the natural response expressions, $A_1$ & $A_2$.

Since we have 2 unknowns, we will need 2 initial conditions. For a typical problem, if asked to find $v(t)$, for $t > 0$ you will need $v_c(0)$, $\frac{dv_c}{dt}$. For $i(t)$ find $i(0)$, $\frac{di}{dt}$. To find $\frac{dv_c}{dt}$ use

\[\frac{dv_c(t)}{dt} = \frac{i_c}{C} \bigg|_{t=0^+}\]

for $\frac{di_c(0)}{dt}$ use

\[\frac{di_c(t)}{dt} = \frac{V_c}{L} \bigg|_{t=0^+}\]
we have already found \( i_c(0^+) \) and \( v_c(0^+) \) so these next initial conditions are easy

\[
\begin{align*}
\frac{dv_c(0^+)}{dt} &= \frac{i_c(0^+)}{C} = \frac{0.28\text{m}}{1\mu} = 278\text{V/s} \\
\frac{di_L(0^+)}{dt} &= \frac{v_c(0^+)}{L} = \frac{0}{L} = 0 \text{A/s}
\end{align*}
\]

Finally, find \( \frac{di_L(0^+)}{dt} \) for practice purposes.

What do we know about \( i_L(t) \)? Using KCL, we know \( i_r = i_L + i_c \).

We can also see if \( \frac{d}{dt} \text{(KCL)} \) which = \( \frac{d}{dt} (i_r = i_L + i_c) \)

is useful. This gives us \( \frac{di_L}{dt} = \frac{di_L}{dt} + \frac{di_c}{dt} \)

So if we can find \( \frac{di_L}{dt} \) AND we can find \( \frac{di_c}{dt} \)

Then we are done.

For \( i_R \), using KVL we have 80 = \( i_R R + v_c \)

\( \frac{d}{dt} \text{(KVL)} \Rightarrow 0 = R \frac{di_R}{dt} + \frac{dv_c}{dt} \) so \( \frac{di_R(0^+)}{dt} = -\frac{1}{R} \frac{dv_c(0^+)}{dt} = -\frac{278\text{V/s}}{45\Omega} \)

At last we have

\[
\frac{di_c(0^+)}{dt} = \frac{di_R(0^+)}{dt} - \frac{di_L(0^+)}{dt} = -6.17 - 0 = -6.17\text{mA/s} = \frac{di_c(0^+)}{dt}
\]
**Final Conditions**

Now we assume $t \to \infty$, so the capacitor behaves as an open circuit again, and the inductor as a short circuit.

We have

![Circuit Diagram]

As for the initial conditions, with the inductor as a short circuit, we know $V_L = 0$. Since $C \& L$ are in parallel, we know $V_C = V_L \Rightarrow V_C(\infty) = 0 \text{ V}$

$I_L: \quad i_L(\infty) = \frac{80}{25k + 20k} = 1.78 \text{ A} = i_L(\infty)$