Chapter 8 Practice Initial + Final Values prob

$t < 0$

10V(t)V

Find initial values: \( v(0^-) \), \( i(0^-) \), \( \frac{dv}{dt}(0^-) \), \( \frac{di}{dt}(0^-) \)

final values: \( v(0^+) \), \( i(\infty) \)

Things you know are always true

in the moment of time when something in the circuit switches on or off it is always true that

\[ i_L(0^-) = i_L(0^+) \] (assuming change occurs at time \( t=0 \))

\[ v_C(0^-) = v_C(0^+) \]

From these continuity relationships we can deduce other voltages, currents, \( \frac{di}{dt} \) and \( \frac{dv}{dt} \)

we also use Ohm's law, KVL & KCL which also are always true

Time \( t=0^- \) For this type of problem always assume the circuit has been in its initial state long enough for all \( C + L \) to be fully charged, (state)

\[ i_L = \frac{10}{3+5} = \frac{10}{8} A \]

\[ v_C = v_{\infty} = \left( \frac{10}{8} \right) - \left( \frac{5}{8} \right) = \frac{50}{8} = \frac{25}{4} V \]
For $t = 0^-$, we have $i_L = 1.25A$
$V_c = 6.25V$

Therefore, from continuity of $i_L + V_c$ from one instant to the next, we know

\[ i_L(0^+) = 1.25A \]
\[ V_c(0^+) = 6.25V \]

Initial conditions of the circuit at time $t = 0$

**DEDUCE** the remaining $i$ & $V$ @ $t = 0$

\[ V_{5R} = V_c = 6.25V \]
For the 5Ω resistor, if $V = 6.25V$, then $i = \frac{6.25}{5} = 1.25A$

\[ i_{3R} = i_L = 1.25A \]

Again using Ohm's Law

\[ V_{3R} = 1.25(3) = 3.75V \]

Note also that w/ KVL

\[ 10V \text{ (source)} = V_{3R} + V_{5R} = 3.75 + 6.25 \]

Also

\[ 10V = V_{3R} + V_c = 3.75 + 6.25 \]
\[ w/ V_L = 0 \]

**ON TO THE FIRST DERIVATIVES**

\[ \frac{dV_c}{dt} = \frac{1}{C} i_c \]
\[ \frac{di_L}{dt} = \frac{1}{L} V_c \]

\[ \star \text{ We are interested in the state of the circuit at time } t = 0^+ \]

This means the 10V source is off and the 1A source is now on
\( t = 0^+ \)

Using KCL we have
\[ i_L + 1A = i_c + i_{5Ω} \]

we know \( i_c(0^-) = 1.25A \), and from above \( i_{5Ω}(0^-) = 1.25A \)

so \( i_c = 1A \)

then
\[ \frac{dv_c}{dt} \bigg|_{t=0^+} = \frac{1}{(0.1)}(1) = \frac{10V}{s} \]

For \( \frac{di_c}{dt} \) we need to find \( V_L \)

with the 10V source turned off (so \( V_L = 0V \)) we have

in the left loop
\[ V_{5Ω} + V_L + V_c = 0 \]

we found \( V_{3Ω}(0^+) \) above
\[ 3.75 + V_L + 6.25 = 0 \]

\( V_L = -10V \)

\[ \therefore \frac{di_c}{dt} \bigg|_{t=0^+} = \frac{1}{L} V_L = \frac{1}{4}(-10) = -40mA/s \]

For \( t = \infty \)

we see there is a simple current divider with the 1A source dividing through the 5Ω and 3Ω resistors

\( i_L = -i_{3Ω} = -1 \left( \frac{5}{8} \right) = -0.625mA = i_L(\infty) \)

\[ \therefore i_{5Ω} = 1 \left( \frac{3}{8} \right) = 0.375mA. \text{ Finally } V_c(\infty) = V_{5Ω}(\infty) = 5(0.375) = 1.875V \]