Class 6. Review Mesh & Nodal
(introduce Linearity & Sup)

Practice Circuit Analysis 1

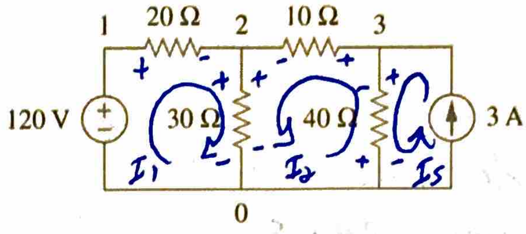
Mesh

- Find all currents and voltages

KVL loop 1 $\Sigma V_{loop} = 0$

$-120 + V_{20\Omega} + V_{30\Omega} = 0$

① $120 = 20(I_1) + 30(I_1 + I_2)$
$= 50 I_1 + 30 I_2$



loop 2 $V_{40\Omega} + V_{10\Omega} + V_{30\Omega} = 0$

$40(I_2 - \cancel{I_s}) + 10(I_2) + 30(I_1 + I_2)$
3

② $+120 = 30 I_1 + 80 I_2$

loop 3 $I_s = 3$ ← loop current is given ★ cannot write KVL for this loop

- Find all currents and voltages

①-② = 0 = $20 I_1 - 50 I_2$

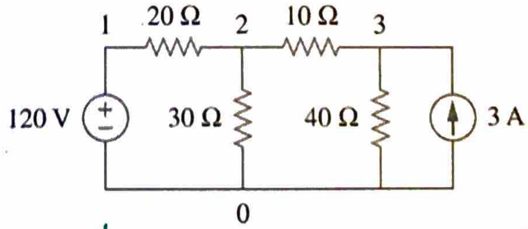
$I_1 = \frac{5}{2} I_2$

→① $120 = 50\left(\frac{5}{2} I_2\right) + 30 I_2$

$= 125 I_2 + 30 I_2$

$= 155 I_2$

$I_2 = 0.77A$ ∴ $I_1 = 1.94A$



Node Voltages? (for comparison)

$V_2 = 30(I_1 + I_2) = 81.3V$

$V_3 = 40(I_s - I_2) = 89.2V$

Nodal Practice Circuit Analysis 1

- Find all currents and voltages

KCL @ 2 $\quad \Sigma I_{in} = \Sigma I_{out}$

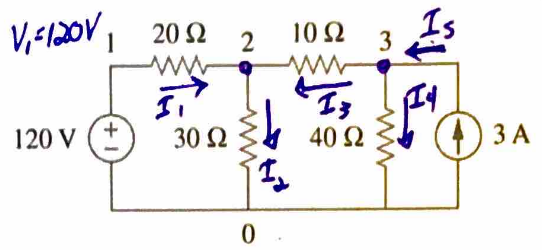
$$I_1 + I_3 = I_2$$

$$6\left(\frac{V_1 - V_2}{2\cancel{0}} + \frac{V_3 - V_2}{1\cancel{0}} = \frac{V_2 - 0}{3\cancel{0}}\right)6$$

$$3V_1 - 3V_2 + 6V_3 - 6V_2 = 2V_2$$

$$3(120) - 9V_2 + 6V_3 = 2V_2$$

$$360 = 11V_2 - 6V_3$$

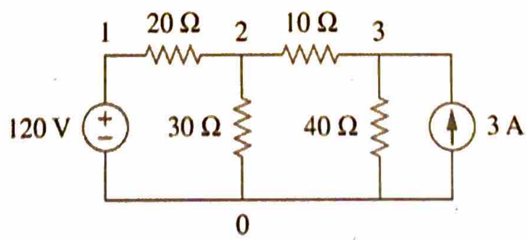


KCL @ 3 $\quad I_s = I_3 + I_4$

$$40\left(3 = \frac{V_3 - V_2}{1\cancel{0}} + \frac{V_3 - 0}{4\cancel{0}}\right)40$$

$$120 = 4V_3 - 4V_2 + V_3 = -4V_2 + 5V_3$$

- Find all currents and voltages



we can write this as

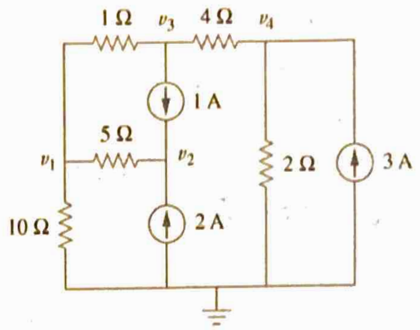
$$\begin{pmatrix} 11 & -6 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 360 \\ 120 \end{pmatrix}$$

and solve using a computer to find $\vec{V} = \begin{pmatrix} 81.3 \\ 89.0 \end{pmatrix}$ so $V_2 = 81.3\ V$

$V_3 = 89.0\ V$

Discussion Circuit Analysis

- How can you find the voltages indicated?
- Compare ability to use nodal analysis vs. mesh analysis.

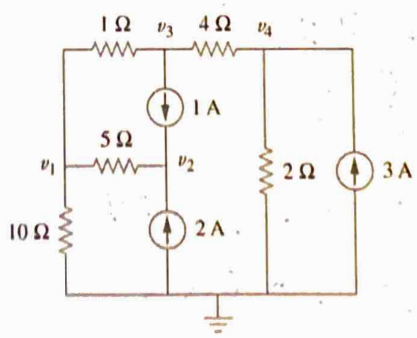


Can do nodel with KCL eq'ns

Cannot do mesh, w/ KVL eq'ns
Voltage across 1A source & 2A source ??

We need to substitute Ohm's Law into the KVL equations, using net I and R for each element. But there is NO WAY to know the resistance for a current source. Ohm's Law is defined for resistors, not for sources. So we cannot use Mesh analysis when more than one mesh, or loop, shares a current source.

- How to find the voltages indicated?



Practice Circuit Analysis 2

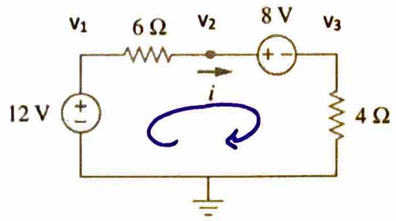
KVL

- Find all currents and voltages

$-12 + V_{6\Omega} + 8 + V_{4\Omega} = 0$

$4 = V_{6\Omega} + V_{4\Omega} = 6I + 4I$

$4 = 10I$ so $I = 0.4A$



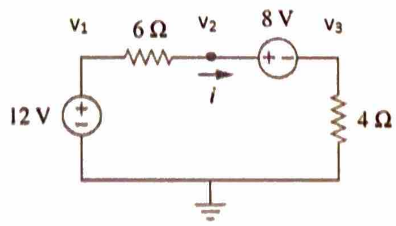
observe $V_1 = 12V$ = Vsource

$V_2 = V_3 + 8$ so $V_2 = 1.6 + 8 = 9.6V$

$V_3 = 4(0.4) = 1.6V$

- Find all currents and voltages

KCL



write KCL for node 2

$\Sigma I_{in} = \Sigma I_{out}$

we cannot know, or write an equation for the current through a voltage source

use $I_{6\Omega} = I_{4\Omega}$

$$\frac{V_1 - V_2}{6} = \frac{V_3 - 0}{4} \Rightarrow 12\left(\frac{12 - V_2}{6} = \frac{(V_2 - 8) - 0}{4}\right)12$$

$24 - 2V_2 = 3V_2 - 24$

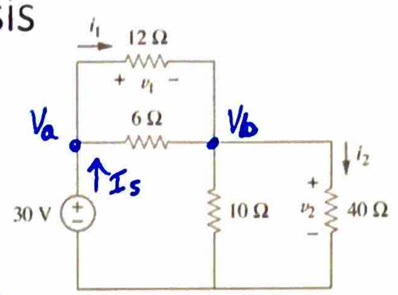
$48 = 5V_2$

$$\boxed{\begin{array}{l} \text{So } V_2 = 9.6V \\ \therefore V_3 = 9.6 - 8 = 1.6V \\ I = \frac{1.6 - 0}{4} = \frac{12 - 9.6}{6} = 0.4A \end{array}}$$

Assume you are asked to find V_1, V_2, i_1 & i_2

Note $V_1 = V_a - V_b$ and $V_2 = V_b$ as labeled

Discuss Practice Analysis



Try nodal analysis

KCL @ a :

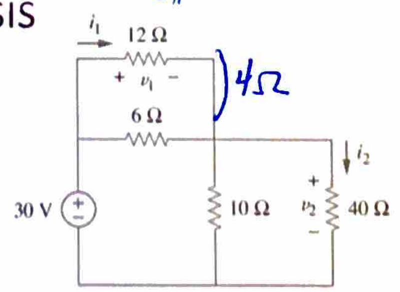
We do not have any expression for I_s, We cannot use nodal analysis

Options ① Mesh analysis w/ 3 loops

② Use R_{eq} for both $\parallel R$ pairs then voltage divider & current divider or simply ohm's law

Discuss Practice Analysis

$6 \parallel 12 = 4\Omega$



$10 \parallel 40 = 8\Omega$

$V_1 = V_S\left(\frac{4}{12}\right) = 10V$

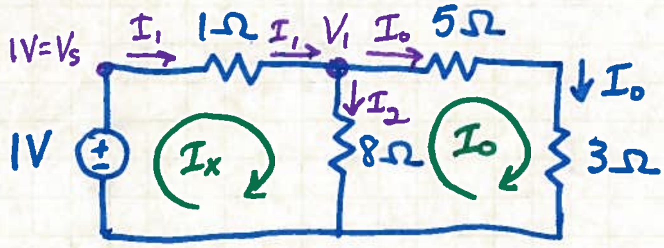
$V_2 = V_S\left(\frac{8}{12}\right) = 20V$

Then with ohm's law

$I_2 = \frac{V_2}{R} = \frac{20V}{40\Omega} = \frac{1}{2}A$

$I_1 = \frac{10V}{12\Omega} = \frac{5}{6}A$

Review Class Problem



⇒ find I_o using different analysis methods

Nodal Analysis (purple labels)

There is one unknown nodal voltage, V_1, so we will have one equation

KCL : $I_1 = I_2 + I_o \Rightarrow \frac{1 - V_1}{1\Omega} = \frac{V_1 - 0}{8\Omega} + \frac{V_1 - 0}{(5+3)\Omega} \Rightarrow 8 - 8V_1 = 2V_1$

$\therefore 8 = 10V_1$ or $V_1 = 0.8V$

Nodal analysis solves for node voltages, Use Ohm's law to find I_o

$I_o = (V_1 - 0)/(5+3) = \frac{0.8}{8} = \boxed{0.1A}$

Mesh analysis (green loop labels) - 2 loops → 2 unk → 2 eq'ns

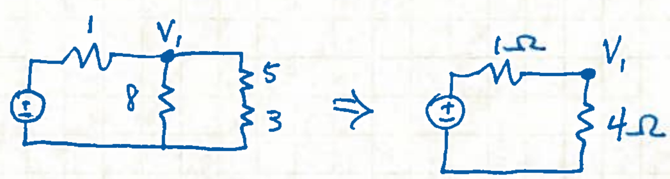
KVL 1 : $-1 + 1(I_x) + 8(I_x - I_o) = 0 \Rightarrow 9I_x - 8I_o = 1$

KVL 2 : $8(I_o - I_x) + (5+3)(I_o) = 0 \Rightarrow 16I_o = 8I_x$ or $I_x = 2I_o$

subs ② into ① $\Rightarrow (18-8)I_o = 1$ or $\boxed{I_o = 0.1A}$

V-divider + Req

$8 \| (5+3) = 4\Omega$



V_1 is the voltage across both the 8Ω center and $(5+3)\Omega$ branches

V-divider to find $V_1 = 1V\left(\frac{4}{1+4}\right) = 1\left(\frac{4}{5}\right) = 0.8V$ so $I_o = \frac{0.8V}{(5+3)\Omega} = \boxed{0.1A}$

Current Divider

with 2 branches of equal resistance - 8Ω and $(5+3)\Omega$, the source current all flows through the 1Ω R, + then divides evenly between the parallel branches (BECAUSE they are equal resistance)

$I_o = I_s\left(\frac{8}{8+(5+3)}\right) = \left(\frac{1V}{(1+4)\Omega}\right)\left(\frac{8}{16}\right) = \frac{1}{5} \cdot \frac{1}{2} = \boxed{0.1A}$