Practice Circuit Analysis 1

- Find all currents and voltages

**KVL loop 1** \[ \sum V_{\text{loop}} = 0 \]
- \[ -120 + V_{20A} + V_{30A} = 0 \]

\[ 120 = 20(I_1) + 30(I_1 + I_2) = 50I_1 + 30I_2 \]

**Loop 2** \[ \sum V_{\text{loop}} = 0 \]
- \[ 40(I_3 - \frac{I_2}{3}) + 10(I_2) + 30(I_1 + I_2) \]

\[ 120 = 30I_1 + 80I_2 \]

**Loop 3** \[ I_3 = 3 \text{ (loop current is given)} \]
* cannot write KVL for this loop

- Find all currents and voltages

\[ \begin{align*}
  \text{①} - \text{②} & = 0 = 20I_1 - 50I_2 \\
  I_1 & = \frac{5}{3} I_2
\end{align*} \]

\[ \begin{align*}
  \text{→ ①} & \quad 120 = 50(\frac{5}{3}I_2) + 30I_2 \\
 & = 125I_2 + 30I_2 \\
 & = 155I_2 \\
 I_2 & = 0.77A \\
 I_1 & = 1.94A
\end{align*} \]

**Node Voltages? (for comparison)**

- \[ V_2 = 30(I_1 + I_2) = 81.3 \text{ V} \]
- \[ V_3 = 40(I_3 - I_2) = 89.2 \text{ V} \]
Nodal Practice Circuit Analysis 1

Find all currents and voltages

KCL @ 2: \[ \Sigma I_{in} = \Sigma I_{out} \]
\[ I_1 + I_3 = I_s \]
\[ 6 \left( \frac{V_1 - V_2}{20} + \frac{V_3 - V_2}{10} = \frac{V_2 - 0}{3} \right) \]
\[ 3V_1 - 3V_2 + 6V_3 - 6V_2 = 2V_2 \]
\[ 3(120) - 9V_2 + 6V_3 = 2V_2 \]
\[ 360 = 11V_2 - 6V_3 \]

KCL @ 3: \[ I_3 = I_2 + I_4 \]
\[ 40 = \frac{V_3 - V_2}{10} + \frac{V_3 - 0}{4} \]
\[ 120 = 4V_3 - 4V_2 + V_3 = -4V_2 + 5V_3 \]

Find all currents and voltages

we can write this as
\[
\begin{pmatrix}
11 & -6 \\
-4 & 5
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix}
= 
\begin{pmatrix}
360 \\
120
\end{pmatrix}
\]

and solve using a computer

to find \[ \mathbf{V} = \begin{pmatrix} 81.3 \\ 89.0 \end{pmatrix} \] so \[ V_2 = 81.3 \text{ V} \]
\[ V_3 = 89.0 \text{ V} \]
Discussion Circuit Analysis

• How can you find the voltages indicated?

• Compare ability to use nodal analysis vs. mesh analysis.

We need to substitute Ohm's Law into the KVL equations, using net I and R for each element. But there is NO WAY to know the resistance for a current source. Ohm's Law is defined for resistors, not for sources. So we cannot use Mesh analysis when more than one mesh, or loop, shares a current source.

• How to find the voltages indicated?
Practice Circuit Analysis 2

- Find all currents and voltages

\[ -12 + V_{6a} + 8 + V_{4a} = 0 \]
\[ 4 = V_{6a} + V_{4a} = 6I + 4I \]
\[ 4 = 10I \text{ so } I = 0.4A \]

Observe \( V_1 = 12V = V_{\text{source}} \)
\[ V_2 = V_3 + 8 \]
\[ V_3 = 4(0.4) = 1.6V \]

\[ V_2 \] is 1.6 + 8 = 9.6 V

- Find all currents and voltages

KCL

Write KCL for node 2
\[ \Sigma I_{\text{in}} = \Sigma I_{\text{out}} \]

We cannot know, or write an equation for the current through a voltage source.

Use \( I_{6a} = I_{4a} \)
\[ \frac{V_1 - V_2}{6} = \frac{V_3 - 0}{4} \]
\[ \Rightarrow \frac{12 - V_2}{6} = \frac{(V_2 - 8) - 0}{4} \]
\[ 24 - 2V_2 = 3V_2 - 24 \]
\[ 48 = 5V_2 \]
\[ \text{so } V_2 = 9.6V \]
\[ \therefore V_3 = 9.6 - 8 = 1.6V \]
\[ I = \frac{1.6 - 0}{4} \cdot \frac{12 - 9.6}{6} = 0.4A \]
Assume you are asked to find $V_1, V_2, i_1, i_2$
Note $V_1 = V_a - V_b$ and $V_2 = V_b$ as labeled

Discuss Practice Analysis

Try nodal analysis

KCL at a:
We do not have any expression for $I_s$, we cannot use nodal analysis

Options
1. Mesh analysis w/ 3 loops
2. Use Req for both IIR pairs
then voltage divider & current divider
or simply ohm's law

$V_1 = V_s \left(\frac{4}{12}\right) = 10V$
$V_2 = V_s \left(\frac{8}{12}\right) = 20V$

Then with ohm's law
$I_2 = \frac{V_2}{R} = \frac{20V}{40\Omega} = \frac{1}{2}A$
$I_1 = \frac{10V}{12\Omega} = \frac{5}{6}A$

Discuss Practice Analysis

\[\frac{6}{12} = 4\Omega\]
\[\frac{10}{40} = 0.5\Omega\]
**Review Class Problem**

\[ \begin{align*}
1V &= V_s \\
I_1 &= 1 \Omega \, I_1, \quad V_1, \quad I_o \quad 5\Omega
\end{align*} \]

\[ I_0 \quad 3\Omega \]

\[ \Rightarrow \text{find } I_0 \text{ using different analysis methods} \]

**Nodal Analysis (purple labels)**

There is one unknown nodal voltage, \( V_1 \), so we will have one equation.

\[ \text{KCL: } I_1 = I_2 + I_0 \Rightarrow \frac{1 - V_1}{1\Omega} = \frac{V_1 - V_0}{8\Omega} + \frac{V_1 - V_0}{(5+3)\Omega} \Rightarrow 8 - 8V_1 = 2V_1 \]

\[ \Rightarrow 8 = 10V_1 \quad \therefore \quad V_1 = 0.8V \]

Nodal analysis solves for node voltages, use Ohm's Law to find \( I_0 \):

\[ I_0 = \frac{V_1 - V_0}{(5+3)} = \frac{0.8V}{8} = 0.1A \]

**Mesh analysis (green loop labels)** - 2 loops \( \Rightarrow \) 2 unk \( \Rightarrow \) 2 eq's

\[ \text{KVL 1: } -1 + 1(I_x) + 8(I_x - I_0) = 0 \Rightarrow 9I_x - 8I_0 = 1 \]

\[ \text{KVL 2: } 8(I_0 - I_x) + (5+3)(I_0) = 0 \Rightarrow 16I_0 = 8I_x \quad \text{or} \quad I_x = 2I_0 \]

Subs (2) into (1) \( \Rightarrow (18 - 8)I_0 = 1 \quad \therefore \quad I_0 = 0.1A \]

**V-divider & Reg**

\[ \frac{8}{(5+3)} = 4\Omega \]

\[ V_1 \quad \text{is the voltage across both the } 8\Omega \text{ center and } (5+3)\Omega \text{ branches} \]

\[ V_\text{divider to find } V_1 = 1V \left( \frac{4}{1+4} \right) = 1(\frac{4}{5}) = 0.8V \quad \text{so} \quad I_0 = \frac{0.8V}{(5+3)\Omega} = 0.1A \]

**Current Divider**

with 2 branches of equal resistance - \( 8\Omega \) and \( (5+3)\Omega \), the source current all flows through the 12 \( R \), then divides evenly between the parallel branches (since they are equal resistance),

\[ I_0 = I_s \left( \frac{8}{8+(5+3)} \right) = \left( \frac{1V}{(1+4)\Omega} \right) \left( \frac{8}{16} \right) = \frac{1}{5} \cdot \frac{1}{2} = 0.1A \]