Frequency Response: Transfer Functions & Bode Plots

EGR 220, Chapter 14.1–14.4
April 14, 2016

Overview

• Frequency Response of a circuit
• The DeciBel – a logarithmic unit
• Transfer function
  o Mathematical representation of frequency response
• Bode plot
  o Graphical representation of frequency response

Frequency Response Motivation

• Discussion – examples of filters?
• Electrical circuit filters
  o Low- and High-pass
  o Resonant circuits
  o Passive and active filters
• Circuits behave differently for different input voltage (or current) frequency
  o Some frequencies passed through (amplified or not)
  o Some frequencies attenuated or blocked

Resonant Circuits

• Unique characteristics are __________?
  1)  
  2)  
  3)  
• How does this property allow for ‘frequency discrimination’?
• How does this property allow the circuit to be ‘tuned’?
Logarithms

- What is a logarithm?
- Plotting (lab 7) data – linear v. log scale?

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$\log_{10}$?</th>
<th>Amplitude</th>
<th>$\log_{10}$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.0013</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>7,500</td>
<td>0.020</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>15,000</td>
<td>0.042</td>
<td>0.020</td>
<td></td>
</tr>
<tr>
<td>30,000</td>
<td>0.206</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>60,000</td>
<td>3.080</td>
<td>0.206</td>
<td></td>
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<td>4.250</td>
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<td>0.042</td>
<td></td>
</tr>
<tr>
<td>250,000</td>
<td>0.90</td>
<td>0.020</td>
<td></td>
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</table>

Decibel & H(s) Magnitude

$|H_{dB}| = 20 \log_{10} |H(j\omega)|$

- A logarithmic unit
- Factor of ‘20’ ...
  - ‘Deci’ → multiplication by a factor of 10
  - Based on a ‘power’ transfer function
  - $P = V^2/R = |R| \Rightarrow \text{‘square’ leads to multiplication by a factor of 2 when using logarithm}$
- Bode plots use logarithm & exponential form
  - Multiplication of terms becomes addition
  - Division of terms becomes subtraction

Determine dB value of each number

<table>
<thead>
<tr>
<th>Specific gain and their decibel values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude H</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>0.001</td>
</tr>
<tr>
<td>0.01</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>$1/\sqrt{2}$</td>
</tr>
<tr>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>1000</td>
</tr>
</tbody>
</table>
Logarithms – Using dB

• Plotting (lab 7) data – linear v. log scale?

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<tr>
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<th>$\log_{10}(\omega)$</th>
<th>Amplitude</th>
<th>Decibel (dB)</th>
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<tr>
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Frequency Response

• The frequency response of a circuit is a representation of circuit behavior from input $\omega = 0$ to $\omega = \infty$

1. **Mathematical** representation
   - Transfer function
2. **Graphical** representation
   - Bode Plot
     - Amplitude response (to changes in frequency)
     - Phase response (to changes in frequency)
Transfer Functions

\[ H(s) = \frac{N(s)}{D(s)} = \frac{\text{Output}}{\text{Input}} \]
(with \( s = j\omega \))

- If output = \( V_s \), input = \( I_s \) then \( H(s) = ? \)
- What other forms can this function, \( H(s) \), take?

Key Concept:
Capacitor & Inductor Behavior

- Capacitor Impedance at \( \omega = 0 \)
- Capacitor Impedance at \( \omega = \infty \)
- Inductor Impedance at \( \omega = 0 \)
- Inductor Impedance at \( \omega = \infty \)

Frequency Response → Frequency Dependence

- What is the input impedance of these circuits if
  - \( f = 0 \) Hz
  - \( f = 60 \) Hz
  - \( f = \infty \)

RC Frequency Response

\[ H(s) = \frac{V_o}{V_s} = ? \]
Circuit behavior changes with changing input frequency

How does it change? What is it that changes?

How do we represent this changing behavior?

Application: Speaker System
New Tool: Bode Plot

- Step 1: Convert expression for $H(s)$ to standard form (in text; following slides)
  - Uses units of decibels
- Step 2: Find “zeros” and “poles,” special frequencies that make
  - Zeros: $H(s) =$
  - Poles: $H(s) =$
- Step 3: Plot the amplitude and phase over all frequencies, using heuristics
  - Observations of various frequency responses give us shortcuts for graphing

### Lab 7 Transfer Function

$$H(s) = \frac{V_C}{V_s} = \frac{1/sC}{R + sL + 1/sC} = \frac{1}{sRC + s^2LC + 1}$$

$$= \frac{1}{s^2LC + sRC + 1}$$

$$= \frac{K_1}{(s + p_1)(s + p_2)} = \frac{K_2}{(1 + s/p_1)(1 + s/p_2)}$$

### Lab 7 RLC Circuit Frequency Response

$R = 2\Omega$
$L = 47\text{mH}$
$C = 100\text{pF}$
% Find the transfer function & Bode plot for Lab 7 RLC circuit
% Define the circuit parameters
R = 2000;
L = 47E-3;
C = 100E-12;

% Define the transfer function and roots of the RLC circuit
Hs1 = tf([1], [L*C, R*C, 1])
rs1 = roots([L*C, R*C, 1])
Hs2 = tf([inv(L*C), 1, R/L, inv(L*C)]) % tf in standard form
rs2 = roots([1, R/L, inv(L*C)])

% Plot the Bode plot using the built-in Matlab function (note
% you cannot use this for the lab memo and plots)
bode(Hs1)
bode(Hs2)

Hs1 =
\frac{1}{4.7e-12 s^2 + 2e-07 s + 1}

rs1 = 1.0e+05 *
-0.2128 + 4.6077i
-0.2128 - 4.6077i

Hs2 =
\frac{2.128e11}{s^2 + 4.255e04 s + 2.128e11}

rs2 = 1.0e+05 *
-0.2128 + 4.6077i
-0.2128 - 4.6077i

**Frequency vs. Time Domain**

- **Time domain analysis**
  1) Determine system state at \( t = 0 \), \( t = \infty \)
  2) Determine how the system transitions from time \( = 0 \) to \( \infty \)
  3) Determine which moments in time are 'special' (\( \tau \), settling time for 2\( \text{nd} \) order)
  4) Input: step input, leads to 'step response'
**Frequency vs. Time Domain**

- Frequency domain analysis
  1) Determine system state at $\omega = 0$, $\omega = \infty$
  2) Determine how the system transitions from frequency $= 0$ to $\infty$ (output amplitude and phase shift)
  3) Input: sinusoidal input, leads to transfer function and the ‘frequency response’
  4) Determine which frequencies are special (zeros & poles)
  5) The transfer function is a complex number function, so analysis includes amplitude and phase

**For Next Class**

- Read section 14.4 before coming to class – probably more than once
- We will very briefly review the highlights of section 14.4 in class
- Class time will be for doing problems together

**Special $\omega = \text{Zeros & Poles}$**

$$H(s) = \frac{N(s)}{D(s)} = \frac{200j\omega}{(j\omega + 2)(j\omega + 10)} = \frac{200s}{(s + 2)(s + 10)}$$

- Mathematically, how do we find the zeros and poles?
- In terms of algebra,
  - Zero: $H(s) = 0$ when ____________?
  - Pole: $H(s) = \infty$ when ____________?

**Summary**

- Transfer function, $H(\omega)$
  - ‘Impedance’ is one possible transfer function
  - Other $H(\omega)$ are admittance, voltage gain and current gain
- Bode plot
  - Phasors have an amplitude and phase
  - Bode plots represent all possible values for a given amplitude and phase over all frequencies