Sinusoidal Steady-State: Electric Power

EGR 220, Chapter 11
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Complex Power & the Power Factor, \( pf \)

- \( S = P + jQ \) (recall the trig identity with \( v(t)^* i(t) \))
  \[ S = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + j \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) \]

- \( pf \) defined as the cosine of the angle between \( V \) and \( I \)
  - The power factor, \( pf = \cos(\theta_v - \theta_i) \)
  - Leading power factor means current leads voltage

SSS: Electric Power Systems

- Power:
  - Complex power, \( S = P + jQ \)
  - The ‘power factor’
  - “Real” power vs. reactive power
- Root Mean Square: RMS values
- Power transfer \( \to \) maximum possible

The Power Factor from Phasors

- Complex power, \( S = P + jQ = S<\theta_z^\circ \)
  \[ P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \]
  \[ V = V_m \angle \theta_v \]
  \[ I = I_m \angle \theta_i \]
  \[ \frac{1}{2} VI = \]
  \[ \frac{1}{2} VI^* = \]
Discussion - Important Angles:

- What is the connection between
  1. Phase angle of impedance, \( Z \)
  2. Phase difference between \( V \) & \( I \)
  3. Relative values of \( P \) & \( Q \)

Complex Power: The Power Triangle

\[
Z = R + jX
\]

\[
\frac{V}{I} = \frac{V_m}{I_m} \angle \theta
\]

Example: Power Factor

- Find the power factor and state if it is leading or lagging,

Check RMS at an Outlet

\[
X_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} x^2 dt} = \frac{X_m}{\sqrt{2}}
\]
RMS: Root Mean Square

- RMS is also referred to as the effective value
- The effective value of an ac signal IS the dc value that delivers the same average power to a resistor

Maximum Average Power Transfer

- For circuits with complex impedance, when does maximum power transfer occur?

\[ Z_L = R_L + jX_L = \]

RMS = Root Mean Square

- Most electrical equipment has two ratings
  - Maximum power, energy, current, voltage...
  - Average or sustained power, energy, current...
- Maximum value
  - Instantaneous value \( \Rightarrow \) time domain
- Average value
  - Phasor domain
  - RMS \( \Rightarrow \) a new ‘average’ measurement (the effect of the AC signal is equivalent to a DC signal at the RMS value)
Summary

- Power is defined with multiple methods, depending upon the problem at hand
  - Instantaneous power
  - Average power
  - Phasors – amplitude (peak) and phase angle
  - RMS value
  - Complex power and power factor
    - Complex power = real power + reactive power
    - $S = P + jQ$
- Maximum average power transfer important in power systems

Summary: Representing Power

- Know which concepts and terms are time domain vs. phasor domain
- Know the definition and significance of:
  - Instantaneous power
  - Average power
  - RMS vs. maximum value
  - Complex power
    - Real power
    - Reactive power
  - Power factor
  - Compensation (pf correction)