First Order Circuits II: Step Response to Complete Response

EGR 220, Chapter 7 part 2
March 5, 2020

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Contemplating Mathematical Models

- One thing that we must remember is that all the mathematical tools we use are a model intended to describe the observed characteristics of the real world.
- Mother Nature doesn't know anything about any equations
- The real world does what it does and we as engineers develop models to understand and predict what will happen in any given situation.

- Steve Umans

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Overview

- Previous Class: Natural response as found in source-free circuits
  - Time dependent functions $v(t)$ & $i(t)$ behavior in first order circuits (circuits with a single storage element)
- Today: Response to a dc 'step' input = Forced response
  - Input is a switch or unit step, u(t) function

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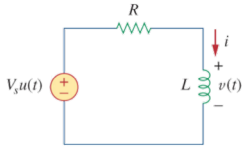
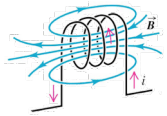
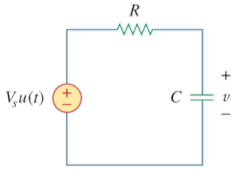
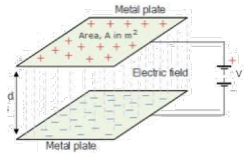
Steady-State Behavior: Behave as… Short Circuit or Open Circuit?









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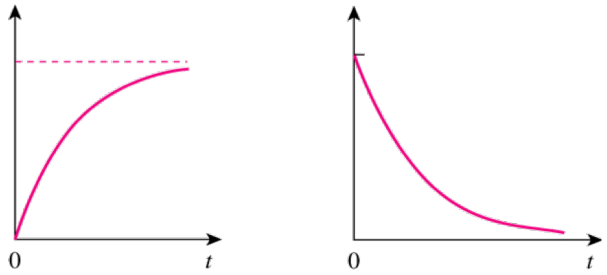
When the Circuit is in DC Steady-State:
Which is Inductor V & I



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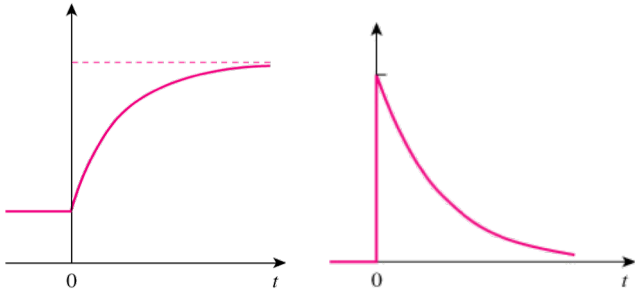
When the Circuit is in DC Steady-State:
Which is Capacitor V & I



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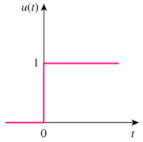
* Continuity Relationship *

- Stored energy cannot change instantaneously → it is "continuous"
- Capacitor: i_c = ----
  - $v_c(0^-) = v_c(0^+) \equiv V_0$
  - Capacitor current?
- Inductor: v_L = ----
  - $i_L(0^-) = i_L(0^+) \equiv I_0$
  - Inductor voltage?



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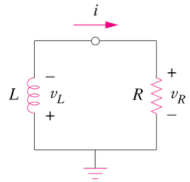
Derive the Natural Response Expression

- Show that $i(t) = I_0 e^{-tR/L}$



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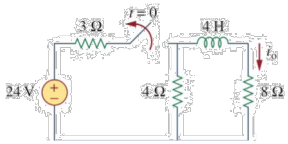
Recap: Natural Response

- Form of solution?
- Time periods of interest?
- *Which values* do you calculate using information from which time periods?

1)

2)



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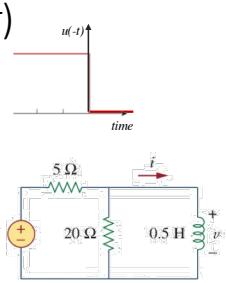
Solve RL Circuit → with $u(-t)$

- What is the role of $u(-t)$?
- Find $i(t)$:
  - Find I_o for t < 0
  - Use $i_L(0^-) = i_L(0^+) \equiv I_0$
  - Find τ for t > 0
    - R = R_{Th} at L
    - 5Ω resistor?



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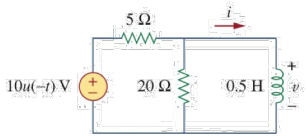
- Write $i(t)$ expression



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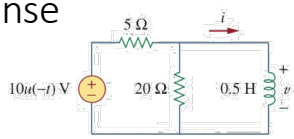
RL Circuit Natural Response

- Write $v_R(t)$ expression



- Discuss polarity of current flow, and $v_R(t)$ and $v_L(t)$

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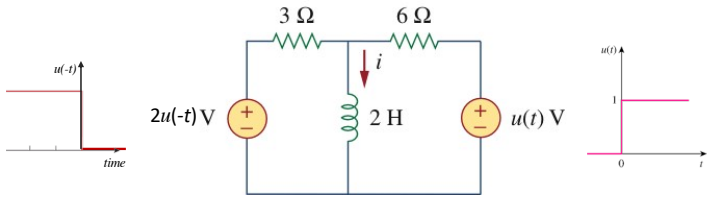
The Complete Response

$$i(t) = i_n(t) + i_f(t)$$

- Complete response = Step response =
  - Natural response (stored energy) +
  - Forced response (independent source)
- The superposition of the response to stored energy & to a power source

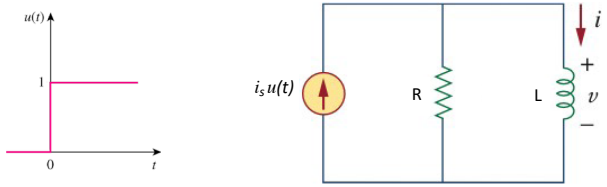
Complete Response of an RL Circuit

- Find $i(t)$ for all time $t > 0$



- But first – to learn the new concept, find only the forced response

RL Circuit: Forced Response



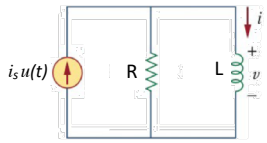
RL Circuit: Forced Response

- Determine $i_L(t)$ and $v(t)$ for all time.
- Assume that the current through the inductor is zero for $t<0$ *(for the forced response, assume no stored energy)*.

1. What is $i_L(t=0)$?
2. What is $v(t=0)$?
3. What is $i_L(t>0)$?

KVL:

KCL:



RL Circuit: Forced Response

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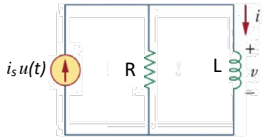

KVL:

KCL:

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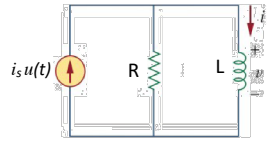
RL Circuit: Forced Response

KVL: $v = L\frac{di_L}{dt} = i_R R$

KCL: $i_L + i_R = i_S \quad \text{so} \quad i_R = -(i_L - i_S)$



Substitute KCL into KVL and rearrange

1)

2)

3)

4)

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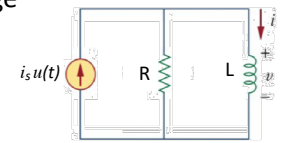
RL Circuit: Forced Response

Substitute KCL into KVL and rearrange



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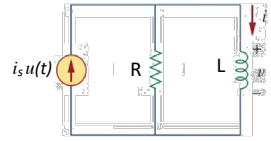
RL Circuit: Forced Response

$$i_L(t) - i_S = -i_s e^{-\frac{R}{L}t}$$



Rearrange to get our desired expressions

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RL Circuit: Forced Response

- What do each of these terms represent?
- What is the graph of this response?
- Note change in notation: i_s to i_∞

$$i_L(t) = i_\infty\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\left\{v_c(t) = V_\infty\left(1 - e^{-t/RC}\right)\right\}$$

Graph of Forced Response?

The Complete Response

$$i(t) = i_n(t) + i_f(t)$$

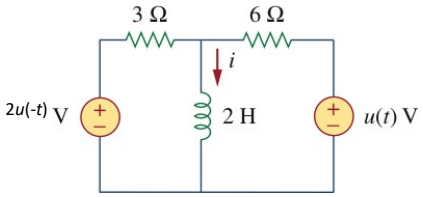
- Complete response = Step response =
  - Natural response (stored energy) +
  - Forced response (independent source)
- The superposition of the response to stored energy & to a power source

Graph of Complete Response?

Complete Response of an RL Circuit

Find *i(t)* for t > 0

1) Write the form of the solution
2) Identify what you need to calculate, and for which time periods



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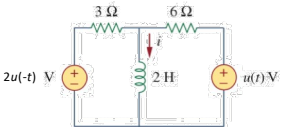
1) Initial Conditions



- Find the initial conditions
  - $t = 0^-$ leads to $t = 0^+$
  - "Continuity relationship" for L and C
- At $t = 0^-$ we know $i_L(0^-)$, so therefore...

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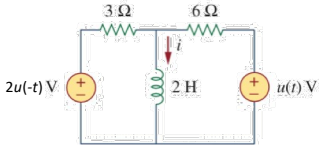
2) Time Constant

- Find $\tau = L/R$
- This often means finding R_{eq} from the storage element



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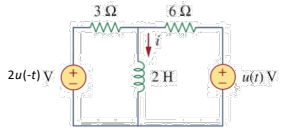
3) Form the Natural Response

$$v_n(t) = V_0 e^{-t/\tau} \quad \text{or}$$

$$i_n(t) = I_0 e^{-t/\tau}$$



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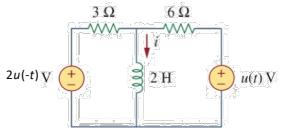
4) Final Condition, I_∞

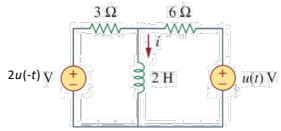
$i_f(t) = i_L(t) = I_\infty\left(1 - e^{-tR/L}\right)$

- Find the value of current at time t = ∞ (again in DC steady-state)



5) Form the Forced Response

$$i_L(t) = I_\infty\left(1 - e^{-tR/L}\right)$$



6) Total, Complete Response

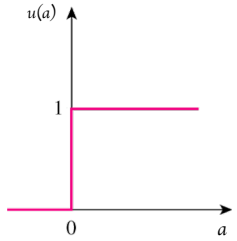
$$i(t) = i_f(t) + i_n(t)$$

$$v(t) = v_f(t) + v_n(t)$$

Text Formulas for Step Response

- RC circuit $v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$
- RL circuit $i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$
- Be careful not to use these equations without understanding how to develop them – you may be asked to explain **each** term

Recap: Unit Step Function



$$u(a) = \begin{cases} 0, & a < 0 \\ 1, & a > 0 \end{cases}$$

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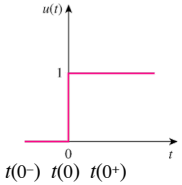
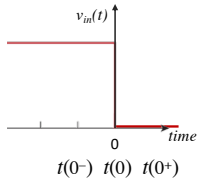
Recap: Unit Step Function



$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

$$u(-t) = \begin{cases} \_\_\_ & t < 0 \\ \_\_\_ & t > 0 \end{cases}$$



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Summary

- Complete response = step response = total response = the sum of
  - Natural response +
  - Forced response
- Practice the analysis method, step by step
- Know what each term means in the i(t) and v(t) step response expressions

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Questions?

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