First Order Circuits I: Source-Free Circuits, the Natural Response

EGR 220, Chapter 7
March 3, 2020

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Overview

- First Order, Source-free circuits
  - One storage element = 1st order circuit
  - Source-free = Natural response
- Analysis method
  - Three time periods of interest
- Solution expression, v(t) and i(t)
  - Time constant
- Examples
- Applets: http://www.falstad.com/circuit/e-index.html

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Capacitors Review

- Stored charge, Q =
- V-I relationship: i_c =
- What does this tell us about the voltage across a capacitor?
  - Think about calculus and taking a derivative
  - What is the steady-state behavior?

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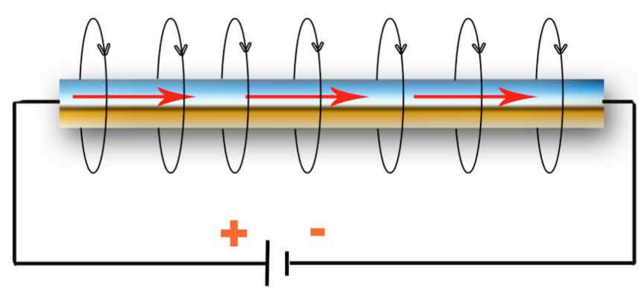
Inductors and Stored B



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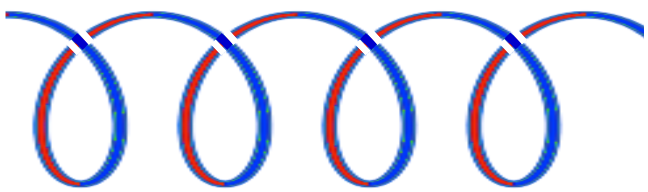
Inductors and Stored **B**



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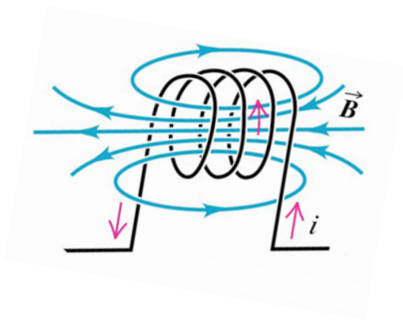
Equivalent Inductance



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* Inductance *

- How is energy stored?
- Property of inductance – experimental observation:
  - A changing current (not a constant DC value)…
  - "induces" a _____________
- V-I expression observed v_L ~ v_L =
- Restrictions on current flowing through an inductor?
  - Think about calculus
  - What is the steady-state behavior?

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The 'Constituent Relations' for R, C and L

- (*i.e.,* What is the V-I relationship?)
- R: V_R = ____________; I_R = _____________
- C: V_c = ____________; I_c = _____________
- L: V_L = ____________; I_L = _____________

  (the dual of the expression for C)

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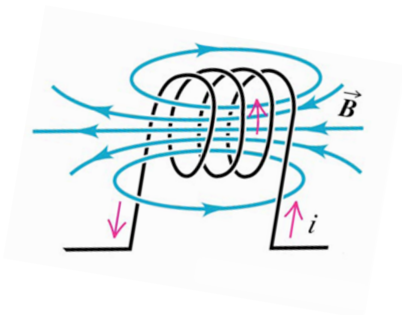
Equivalent Inductance



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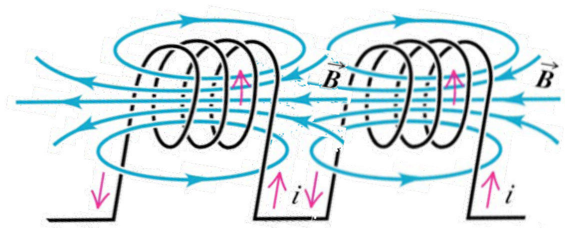
Equivalent Inductance



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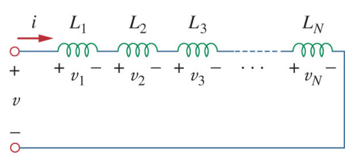
Series Inductors: L_{eq}?



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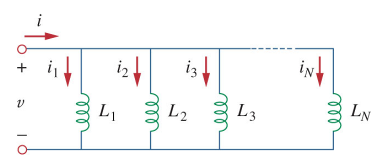
Parallel Inductors: L_{eq}?



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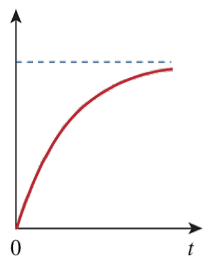
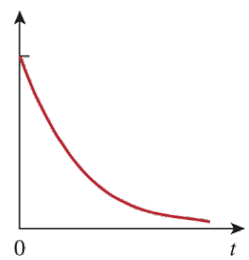
Natural Response of *First Order Circuit*

Charging



Discharging



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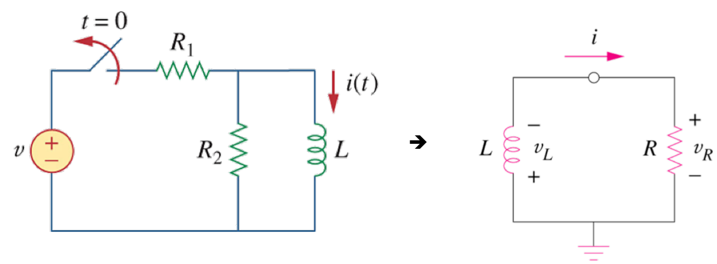
Charging and Discharging



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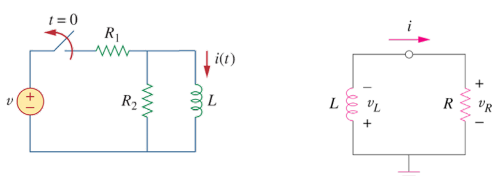
Steady-State Behavior

1) The storage element charges, from a DC power supply
2) Steady-state behavior: After charging "for a long time," the storage element becomes fully charged
   - "For a long time" is defined relative to the ____________
3) The switch is opened and the power supply is disconnected
   - As time goes to infinity...



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Steady-State Behavior

- After charging "for a long time," the storage element becomes fully charged (typically the initial condition).
  - "For a long time" is defined relative to the ____________
- Fully charged behavior is <open/short> circuit:
  - Capacitor: $i_C =$

    Behaves as ________________
  - Inductor: $v_L =$

    Behaves as ________________

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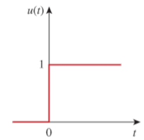
Circuit Response

- Natural response → Initial stored energy
- Forced response → Power source connected
- Complete response → Initial stored energy + connected power source

Also called the 'step response'



(No response at all → No initial stored energy and no power source)

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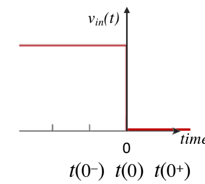
Time periods of interest

DC *input* step source

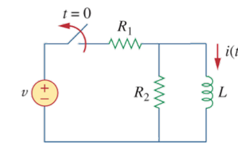


Steady-state 1 → transition → steady-state 2

1)

2)

3)



Assumptions:

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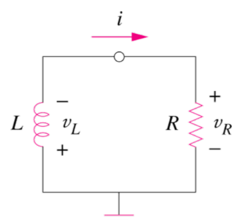
Derive the Natural Response Expression

- Assumptions / **Initial conditions**
  - At t = 0, initial inductor current = _______?
  - At t = 0, R and L voltages are _______?
- Apply KVL in the loop, to find a mathematical expression for $i(t)$

KVL:



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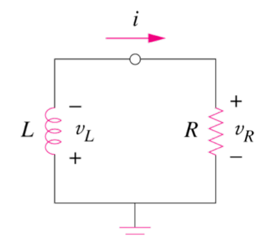
Derive the Natural Response Expression

- Show that $i(t) = I_0 e^{-tR/L}$



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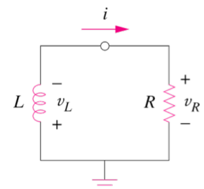

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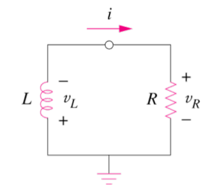
Derive the Natural Response Expression

- With the current equal to:

$$i(t) = I_0 e^{-tR/L} = I_0 e^{-t/\tau}$$



- How do we find $v_R(t)$ and $v_L(t)$?

$$v_R(t) =$$

$$v_L(\mathrm{t}) = -\, v_R(t) =$$

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* Time Constant *

- Time constants for RC and RL circuits are defined as $\tau = RC$ and $\tau = L/R$
- A time constant is the time required for the response ($v(\mathrm{t})$ and $i(\mathrm{t})$) to decay by a factor of ___________ of its initial value

$$v(t) = V_0\, e^{-t/\tau}$$

$$i(t) = I_0\, e^{-t/\tau}$$

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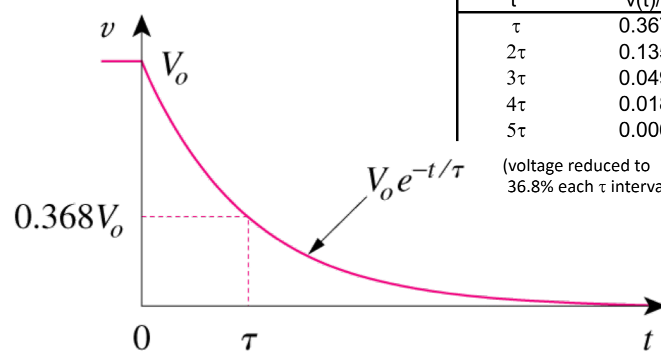
Exponential Response:

Voltage Response of RC Circuit

Table 7.1 Values of v(t)/Vo

t	v(t)/V0
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

(voltage reduced to 36.8% each τ interval)



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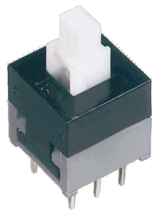
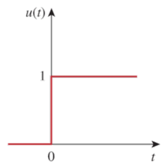
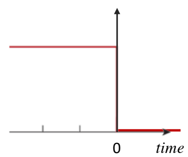
Switches: with Math and in a Circuit Diagram

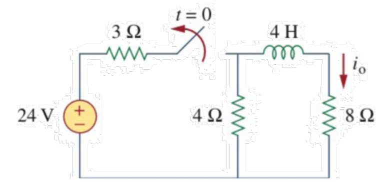






RL Circuit

- Find i(t)
  - I_o for t < 0
  - τ for t > 0, with $R = R_{Th}$ at L



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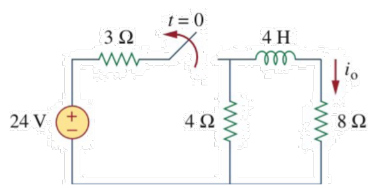

RL Circuit

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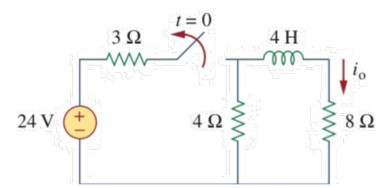
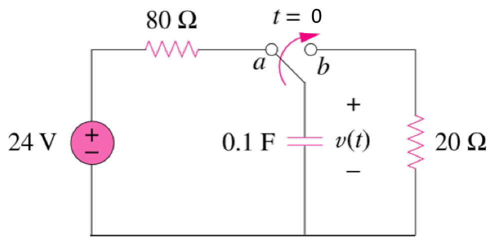


RC Circuit

- At t=0s, the switch is moved from position *a* to *b*.
- ***How*** do we find $v_c(t)$, $i_c(t)$, $i_R(t)$, and $v_R(t)$ at t=10s



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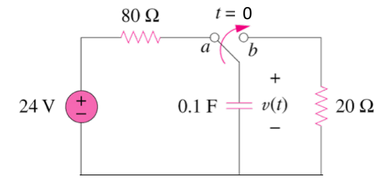
RC Circuit

- How do we find $v_c(t)$, $i_c(t)$, $i_R(t)$, and $v_R(t)$ at t=10s



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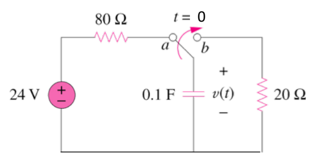

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Lab this week ➢ Time Constants

- Chapter 7 – Use capacitors, not inductors, for lab
- Select R and C values for a time constant that you can use to watch and measure the capacitor charging.
  - Work on charging and then repeat with discharging.
  - Start with the DC power supply and multimeters, and then use the oscilloscope and function generator if you have time and experiments
- Experiment with capacitors
  - (In series with a resistor for equipment safety)
- Think of something you could time and use the RC circuit as a timer.
- Explore why exponential charging and decay explain all the natural dynamic behavior in the universe.

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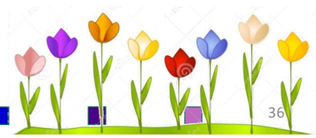
Time Constant

R =
C =
τ =
bucket fills:

R =
C =
τ =
bucket fills:

R =
C =
τ =
bucket fills:



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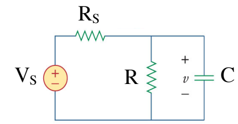
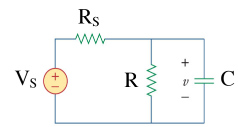
Explore "time constant" $\tau = RC$



Long τ



Short τ

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Lab Memo and Graphs

- Graphs and figures for the memo – read lab handout.
- Be sure to include, and explain, graphs of your circuit behavior in your lab memo.
  - If you will use the oscilloscope this week, then…
  - **Bring a USB drive**, save oscilloscope images and use the saved .png files in your memo
  - (If not this week, then definitely next week)

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Self-Review: Interpret RL & RC Response

- Be comfortable with the answering the following
  - What is the time constant?
  - What is the graph of $i_L(t)$? Of $v_c(t)$
  - What are the graphs of $i_R(t)$, $v_R(t)$, $v_L(t)$ and $i_c(t)$?
  - What happens to the response as L changes?
  - ... As R changes? ... as C changes?
- Be comfortable finding power and energy for these circuits
  - What does the power represent? The energy?

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Summary

- Source-free circuits
  - RC and RL circuits
  - Both are *first order* circuits
- Solving differential equations
  - Exponential and natural logs
- Natural response
- Time constant
- Step functions and switches

Questions?