NOTES: This homework set does two things

1) Reinforce some of the concepts that seemed to still be confusing from exam
   2. These questions go further than those on the midterm, but do have you
   revisit some of those topics.

2) Ask you to go just a bit further than we have gone yet in class, in the final two
   problems, by asking you to identify the poles and/or zeros in the transfer
   function you find, $H(s)$. See page 615 at the beginning of chapter 14 for a
   quick definition of zeros and poles.

**Problem 1: Maximum power transfer**

Let $i_s = 5 \cos (40t)$, $L = 7.5 \text{ mH}$, $C = 40 \text{ mF}$, $R_1 = 12 \Omega$, and $R_2 = 8 \Omega$.

a) Find the load impedance that would allow for the maximum power to be
   transferred to the load. (Note that this is a very important analysis for any
   engineer designing any system to deliver electrical power to a load – you will
   always want to be able to deliver the maximum power possible.)

b) Find the value of this maximum power that is transferred.

Problem 2: Complex power

For the circuit shown, calculate:

a) The power factor
b) The complex power delivered by the source
c) The real power delivered by the source
d) The reactive power delivered by the source
e) What can you do with or to this circuit to make the power factor equal unity?
f) In one or two short phrases, state why you would want unity power factor.
Discussion on Phase Shifting and Frequency Response Analysis

Note that phase shifting is discussed at the end of chapter 9

For the circuit below:

a) Find \( V_o/V_i \) (as a complex algebra expression).
b) Determine the magnitude of this ratio.
c) Determine the phase (i.e., the phase angle) of this ratio.
d) Determine the value of the amplitude at both \( \omega = 0 \) and \( \omega = \infty \).
e) Determine the value of the phase at both \( \omega = 0 \) and \( \omega = \infty \). Note that this phase is the phase difference – or phase shift – between the input and output signals. You can quantify this phase difference if you know the values of \( \omega \), \( R \) and \( C \).

\[
\begin{align*}
I & \quad C \\
\text{+} & \\
V_i & \quad R \quad V_o & \quad \text{+} \\
\text{−} & \\
\end{align*}
\]

a) Using voltage divider

\[
V_o = V_i \left( \frac{R}{R + \left( \frac{1}{j\omega C} \right)} \right) = V_i \left( \frac{j\omega RC}{1 + j\omega RC} \right) \quad \text{...therefore...} \quad \frac{V_o}{V_i} = \frac{j\omega RC}{1 + j\omega RC}
\]

This expression is simply a complex number as we have used in chapter 9 and 10. To find the magnitude and phase, as for polar notation, we need to convert this ratio to polar notation, converting the numerator and denominator separately (as you did for chapter 9 problems)

b) The absolute value bars in math \( |\text{value}| \) are used to indicate ‘magnitude’ or amplitude
\[
\left| \frac{V_o}{V_i} \right| = \left| \frac{j\omega RC}{1 + j\omega RC} \right| = \left( \frac{\omega RC}{\sqrt{1^2 + (\omega RC)^2}} \right)
\]

c) The phase of the numerator and denominator are found, using the fact that \( j \) has a phase of 90°. Again, note that this phase is the phase shift between the input and output.

\[
\angle \left( \frac{V_o}{V_i} \right) = \angle V_o = \frac{\angle 90^\circ}{\angle \arctan (\omega RC)} = \angle (90^\circ - \arctan (\omega RC))
\]

At this point, you may need to review
- basic trigonometry and the values of sine, cosine and tangent for angles from 0° to 360°, and for arctangent which numerical values translate to what angles.
- basic calculus for finding the values of functions as variables approach 0 and \( \infty \).

d) & (e) Substitute \( \omega = 0 \) and \( \omega = \infty \) into the expressions for magnitude and phase above, and determine what happens to each (magnitude and phase) at these boundary frequency conditions.

**Problem 3:**

For the circuit below (assume an input voltage is applied to the left side terminal and the output is taken across the resistor):

a) Find \( \frac{V_o}{V_i} \) (as a complex algebra expression).
b) Determine the magnitude of this ratio.
c) Determine the phase (i.e., the phase angle) of this ratio.
d) Determine the value of the amplitude at both \( \omega = 0 \) and \( \omega = \infty \).
Problem 4:

For the circuit below:
   a) Find the transfer function \( H(s) = \frac{V_o(s)}{V_s(s)} \).
   b) Determine the magnitude of \( H(s) \).
   c) Determine the phase of \( H(s) \).
   d) Identify the poles and zeros for the transfer function \( H(s) \).

Problem 5:

For the circuit below:
   a) Find the transfer function \( H(s) = \frac{V_o(s)}{V_s(s)} \).
   b) Determine the magnitude of \( H(s) \).
   c) Determine the phase of \( H(s) \).
   d) Identify the poles and zeros for the transfer function \( H(s) \).