Chapter 7, Solution 79.

When the switch is in position 1, $i_o(0) = 12/3 = 4A$. When the switch is in position 2,

$$i_o(\infty) = -\frac{4}{5+3} = -0.5\text{ A}, \qquad R_{Th} = (3+5)//4 = 8/3, \qquad \tau = \frac{L}{R_{Th}} = 3/80$$

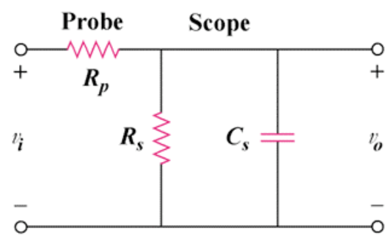
$$i_o(t) = i_o(\infty) + [i_o(0) - i_o(\infty)]e^{-t/\tau} = \underline{-0.5 + 4.5e^{-80t/3}}\ u(t)\text{A}$$

GRAPH: (included at end) The important elements to label on the graph are:

- the axes
- the initial and final values
- the time constant, which is where *i*(t) is at ~37% of its maximum value

Chapter 7, Solution 90.

You can approach this problem as a design problem for the oscilloscope probe → you want to design the 'scope probe, which is to say determine values for R_s and C_s, such that the volage v_o is $0.1v_i$.



Use the voltage divider to find the voltage across R_s, which is equal to v_o. For the *time constant* you know that the equivalent resistance is that from the nodes of the capacitor, and is equal to R_{eq}.

$$v_o = v_i \frac{R_s}{R_s + R_p}, \qquad R_{eq} = R_s \parallel R_p$$

From the design criteria, we know

$$v_o = \frac{1}{10}v_i \longrightarrow \frac{1}{10} = \frac{R_s}{R_s + R_p}$$

Combining these, along with the information in the problem statement:

$$R_s = \frac{1}{9}R_p = \frac{6}{9} = \underline{\mathbf{\frac{2}{3}\ M\Omega}}$$

We are also given that,

$$\tau = R_{eq}C_s = 15\ \mu s$$

Finally, you can solve for C_s: $R_{eq} = R_p \parallel R_s = \frac{6(2/3)}{6+2/3} = 0.6\ M\Omega$

$$C_s = \frac{\tau}{R_{th}} = \frac{15\times10^{-6}}{0.6\times10^{6}} = \underline{\mathbf{25\ pF}}$$

Chapter 8, Solution 17.

Find the initial conditions and α and ω_o, and identify the type of damping (the form of the mathematical expression for the natural response).

$$i(0) = I_0 = 0,\ v(0) = V_0 = 4x15 = 60$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20,\ which\, is > \omega_o.$$

Here we have an overdamped response.

Next find the roots, $s_{1,2}$ and construct the complete response. In this problem, we have a source-free response, so i(∞) = 0A. Thus the complete response is the same as the natural response.

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.68, -37.3$$

$$v(t) = A_1 e^{-2.68t} + A_2 e^{-37.3t}$$

To find a numerical value for dv/dt at t=0, use the fact that i_c = C(dv/dt). *In this circuit* we see that $i_c = i_L$, so:

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = \left.\frac{i_c(t)}{C}\right|_{t=0} = \left.\frac{i_L(t)}{C}\right|_{t=0} = 0\text{V/s}$$

Now use the initial conditions to solve for the constants, A_1 and A_2.

$$v(0) = 60 = A_1 + A_2$$

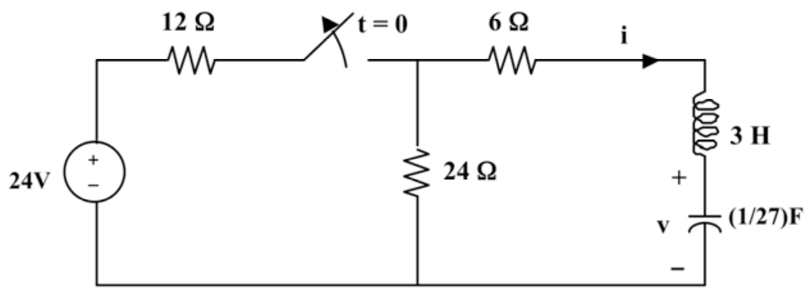
$$\left.\frac{dv(t)}{dt}\right|_{t=0} = 0 = -2.68A_1 - 37.3A_2$$

The final solution is:

v(t) = (64.6e$^{-2.68t}$ – 4.64e$^{-37.3t}$) V

Chapter 8, Solution 21.

By combining some resistors, the circuit is equivalent to that shown below.
$60||(15 + 25) = 24$ ohms.



At t = 0⁻, $i(0) = 0$, $v(0) = 24x24/36 = 16V$
We also can find dv(t)/dt for time t = 0. As above, we find

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = \left.\frac{i_c(t)}{C}\right|_{t=0} = \left.\frac{i_L(t)}{C}\right|_{t=0} = 0\text{V/s} = \frac{dv_c(0)}{dt}$$

** It is **VERY** important to know that it is only in simple series RLC circuits, in this case for $i_c = i_L$ and the open circuit behavior of the capacitor makes $i_L = 0$, that we can use $i_c = 0$. This is often ***not the situation***.

For t > 0, we have a series RLC circuit. R = 30 ohms, L = 3 H, C = (1/27) F

$\alpha = R/(2L) = 30/6 = 5$
$\omega_o = 1/\sqrt{LC} = 1/\sqrt{3x1/27} = 3$, clearly $\alpha > \omega_o$ → overdamped response again

Find the roots:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

Again, with a source-free problem, we have the natural response = complete response, =
$v(t) = [A_1e^{-t} + A_2e^{-9t}]$

Using the initial conditions, solve for A_1 and A_2

$$v(0) = 16 = A_1 + A_2$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = \frac{dv(0)}{dt} = 0 = -9A_1 - A_2$$

To find the final solution: $\boxed{\mathbf{v(t) = (18e^{-t} - 2e^{-9t})\ V}}$

Chapter 8, Solution 24.

When the switch is in position A, the inductor acts like a short circuit so

$$i(0^-) = 12 \text{ A}$$

When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2x10x10x10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x10x10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -5 + \sqrt{25-400} = -5 + j19.365$$

$$i(t) = e^{-5t}\left(A_1 \cos 19.365t + A_2 \sin 19.365t\right)$$

$$i(0) = 12 \text{ A} = A_1$$

$$v = L\frac{di}{dt} \longrightarrow \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = e^{-5t}\left(-5A_1 \cos 19.365t - 5A_2 \sin 19.365t - 19.365A_1 \sin 19.365t + 19.365A_2 \cos 19.365t\right)$$

0= di(0)/dt = $-5A_1 + 19.365A_2$ which leads to $A_2 = (5A_1)/19.365 = 3.098$

Thus,

$$i(t) = \mathbf{e^{-5t}(12\cos(19.365t) + 3.098\sin(19.365t)) \text{ A}}$$

GRAPH: (included at end) The important elements to label on the graph are:

- the axes
- the initial and final values
- the oscillations, with the frequency of ω_d labeled
- the exponential decay of $e^{-\alpha t}$ 'envelope' superimposed upon the oscillatory response

Chapter 8 Problem 49

This problem has a step input for the input function, meaning that you must solve for the ***complete solution***, and not simple for the natural response as for the source-free circuits.

For $t = 0^-$, $i(0) = 3 + 12/4 = 6$ and $v(0) = 0$.

For $t > 0$, we have a parallel *RLC* circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.05) = 2$$

$$\alpha$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.05} = 2$$

Since $\alpha = \omega_o$, we have a <u>critically damped response</u>.

$$s_{1,2} = -2$$

Thus the complete response is of the form:

$$i(t) = I_\infty + [(A + Bt)e^{-2t}], \quad I_\infty = 3$$

Now use the initial conditions to solve for the constants:

$$i(0) = 6 = 3 + A \text{ or } A = 3$$

$$v = Ldi/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$v(0)/L = 0 = di(0)/dt = B - 2x3 \text{ or } B = 6$$

Thus, $\boxed{i(t) = 3 + [(3 + 6t)e^{-2t}] \text{ A}}$

<u>Problem 7: Discuss behavior of 1st & 2nd order, and compare/contrast</u> → options

First Order

- Exponential behavior representing the energy storage element charging and/or discharging energy
- The time constant quantifies the amount of time for charging and discharging

Second Order

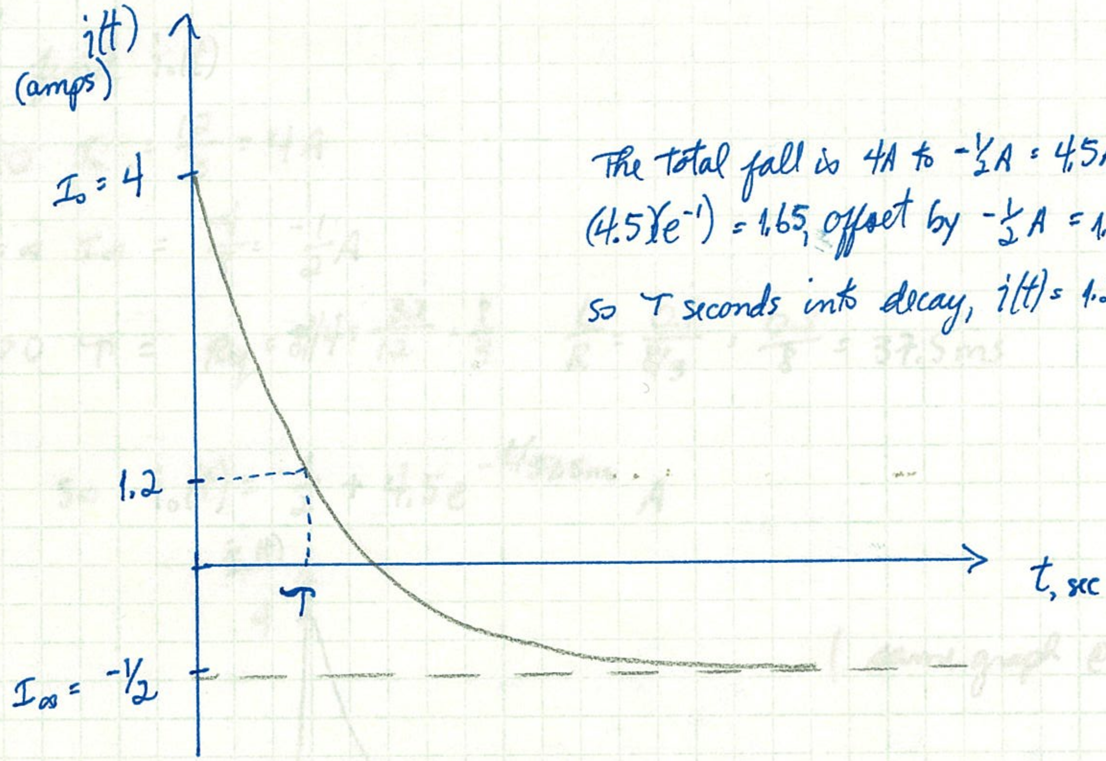
- Circuit behavior representing the two energy storage elements exchanging the stored energy, combining charging and/or discharging behavior
- Represented as a sum of exponentials
- There may be oscillation with two energy storage elements, but the resistance in the circuit may damp it out
- The circuit behavior can be characterized by the natural frequency and the damping coefficient, or equivalently, but the two natural frequencies

Compare 1st and 2nd order

- Both have underlying exponential behavior – single exponential or sum of 2
- Only 2nd order (or higher order) circuits may have oscillatory behavior.
- Both types of circuits have initial conditions, final conditions and transient, or natural behavior to move between these two steady-state conditions

HW 8 graphs

7.79)

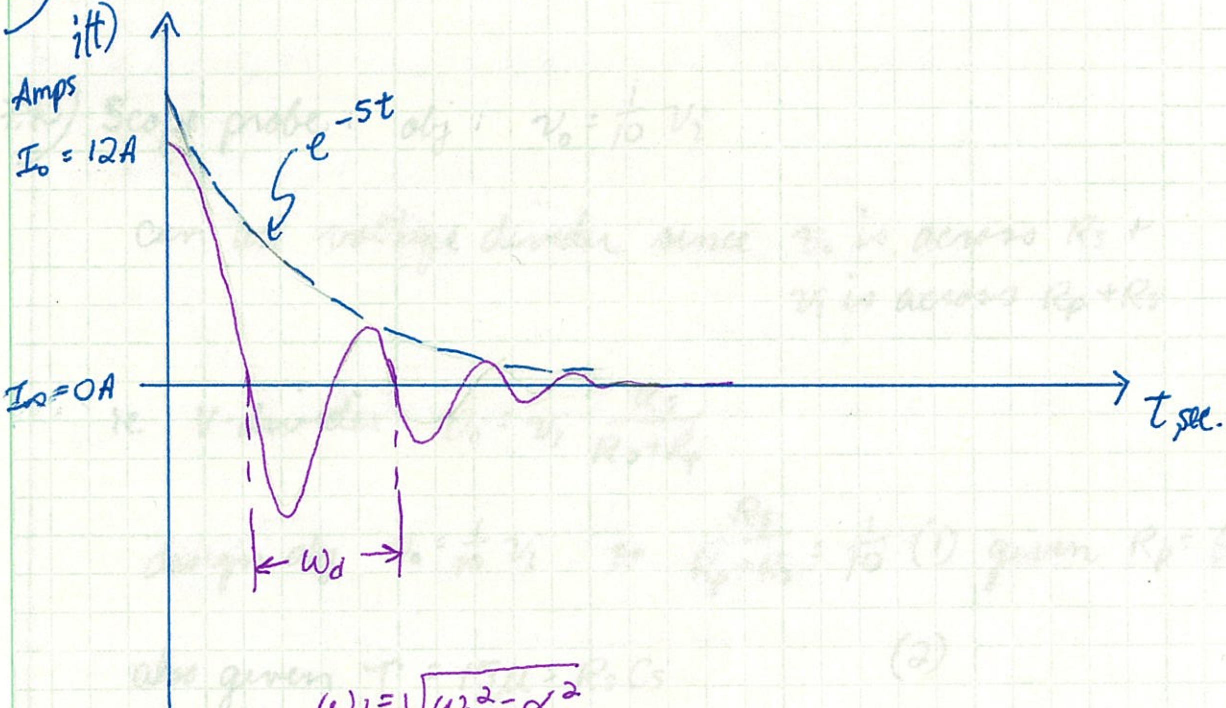


The total fall is 4A to $-\frac{1}{2}A$ = 4.5A

$(4.5)(e^{-1}) = 1.65$, offset by $-\frac{1}{2}A$ = 1.2A

so τ seconds into decay, $i(t) = 1.2A$

8.24)



$\omega_d \equiv \sqrt{\omega_0^2 - \alpha^2}$