1. Find \( v(t) \) for \( t > 0 \)
   - Use \( t < 0 \) to find \( V_0 \)
   - Use \( t > 0 \) to find \( T \)

   We know our solution is of the form \( v(t) = V_0 e^{-t/T} \)

   \( t < 0 \) determines \( V_0 \) → initial condition for \( v(t) \)

   **Always** assume the storage element is fully charged when finding the initial conditions.

   Notice: \( v(t) = V_{2kΩ} \) so use voltage divider

   \[
   V_0 = 40 \left( \frac{2k}{10k + 2k} \right) = 4 \left( \frac{3}{12} \right) = 6.67V = V_0
   \]

   \( t > 0 \) to find \( T \) - we want to know how the voltage across the capacitor changes from time \( t = 0 \) into the future, \( t > 0 \). Therefore find \( T \) for the circuit as it is configured into the future, for \( t > 0 \)

   Here \( T = RC = 2k \cdot 40\mu = 80 ms = T \)

   For time \( t > 0 \) there is no "forcing function" or no independent power source, so there is no energy in the system at time \( t = 0 \). Therefore \( V_0 = 0V \)

   Thus \( v(t) = \text{natural response} = 6.67e^{-t/80ms} = 6.67e^{-12.5t} V \)

   @ time \( t = 80ms \) \( v(t) = 2.45V \)
(2) Source free ckt

since \( v(t) \leq 0 \)

Init. Cond: \( I_0 = 5A \)

Know soln will be

of form
\[
\begin{align*}
v(t) &= V_0 e^{-t/\tau} \\
i(t) &= I_0 e^{-t/\tau}
\end{align*}
\]

\[\text{for } t < 0 \quad I_0 = 5A, \text{ given}\]

\[\text{for } t > 0 \quad \tau = \frac{L}{R} = \frac{0.4}{2 \sqrt{2}} = 0.4 \cdot \frac{1.2}{2} = 0.33 = \frac{1}{3} \text{ s}\]

Source free, so \( I_\infty = 0 \)

Therefore, the soln is
\[
i(t) = 5 e^{-3t} A
\]

For \( v(t) \), we see this is equal to the voltage across \( 3 \Omega \) R

Using current divider
\[
i_2(t) = i(t) \left( \frac{2}{2+3} \right)
\]

\[
\therefore v(t) = 3(i_2(t)) = \frac{6}{5} i(t) = 6e^{-3t} V
\]

For a source-free ckt, this is the natural response
(no forced response here)
This problem introduces the use of a differential equation to describe the behavior of a dynamic circuit. Since this is a first order diff eq, we know it describes a first order circuit → a circuit with a single storage element. Follow the use of this eq'n as described in Section 7.5.

Note: complete response = transient + steady-state response = homogeneous + particular resp.

We anticipate

\[ V_0 > 0 \]
\[ V_{x0} > 0 \]

Given the graph, we see:

\[ V(t) \]

We are given

\[ 4 \frac{dv}{dt} + v = 10, \quad v(0) = 2 \]

For the homogeneous solution, solve

\[ 4 \frac{dv}{dt} + v = 0 \]

\[ \int \frac{dv}{v} = -\frac{1}{4} \int dt \quad \text{which gives us} \quad v_h(t) = Ae^{-t/4} \]

a) from the solution above we see \( T = 4s \)

b) the total sol'n = \( v_h(t) + v_p(t) = Ae^{-t/4} + 10 \)

\[ @ t = \infty, \quad v(t) = 10V \]

c) using the initial condition to find \( A \)

\[ v(0) = 2 = Ae^0 + 10 \quad \text{so} \quad A = -8 \]

\[ \therefore v(t) = 10 - 8e^{-t/4} V \]

And we see graph (a) is the correct form for the solution \( V_0 = 2V, \quad V_{x0} = 10V \)
#3 continued: natural and forced responses

Our solution is of the form $v(t) = 10 - 8e^{-t/4}$

From text equation 7.50 we see

$V_n = V_0e^{-t/\tau}$

$V_f = V_0(1 - e^{-t/\tau})$

Given $v(t) = V_n + V_f$ also $= V_i + V_{ss}$ (eqn 7.51)

$= V_0e^{-t/\tau} + V_0(1 - e^{-t/\tau})$

$= V_0 + (V_0 - V_0)e^{-t/\tau}$ (eqn 7.53)

So for this problem you can observe

$V_0 = 10 \text{ V}$

$V_0 - V_0 = -8$ so $V_0 = 2 \text{ V}$ (This was also found directly in step (c))

So

$v_n(t) = 2e^{-t/4}$ V

$v_f(t) = 10(1 - e^{-t/4})$ V
4) find \( i(t) \) for \( t > 0 \)

First: find \( v(t) \)
and then find \( i(t) \)

\[
V_0 = V_s \left( \frac{3}{6+3} \right) = 60 \left( \frac{3}{9} \right) = 20 \text{ V}
\]

For \( t < 0 \) find \( V_0 \)

\[
V_0 = V_s \left( \frac{3}{6+3} \right) = 60 \left( \frac{3}{9} \right) = 20 \text{ V}
\]

For \( t = \infty \) \( V_4 = 24 \left( \frac{3}{9} \right) = 8 \text{ V} \)

For \( t > 0 \) find \( V_4 \) \( T = RC = 3/6 = 0.5 \) so \( \tau = 2 \cdot 2 = 4 \text{ s} \)

Thus \( v(t) = 8 + (20 - 8) e^{-t/4} = 8 + 12 e^{-t/4} \text{ V} \)

Now \( i_c = C \frac{dv}{dt} = 2 \left[ 8 + 12 \left( \frac{t}{4} \right) e^{-t/4} \right] = -6 e^{-t/4} \text{ A} \)

For current \( i_n = -6 e^{-t/4} \text{ A} \), \( i_f = 0 \)

Though we see \( v_c(t) \) moves from \( v_c(0) = 20 \text{ V} \) to \( v_c(\infty) = 8 \text{ V} \),
the current through the capacitor changes from

\( i_c(0) = -6 \text{ A} \) (a "negative" flow) to \( 0 \text{ A} \)

When the switch changes position, the capacitor begins to discharge, \( i_c(t) \) instantaneously becomes \( -6 \text{ A} \).

\[
\begin{align*}
T = 4.5 s \\
i_c(0) &= -6 \text{ A} \\
i_c(\infty) &= 0 \text{ A} \\
T &= 4.5 s
\end{align*}
\]
Problem 5

Since \( v_s = 10[u(t) - u(t - 1)] \), this is the same as saying that a 10 V source is turned on at \( t = 0 \) and a -10 V source is turned on later at \( t = 1 \). This is shown in the figure below.

For the natural and forced response terms

- The solution from \( 0 < t < 1 \) is a **forced response**
- The solution for \( t > 1 \) is a **natural response** only, since the forcing function has been turned off.

For the plot of this circuit response, see the Matlab script below. The output of this script is the figure below. This figure shows the charging and discharging input pulse response of the RL circuit.

Note that the dotted blue line shows the shape, or form, of the charging response, if it were allowed to reach its full charge. Before the inductor can become fully charged, the input pulse turns off, with the discharging response as shown.
Labeling/Identifying the time constant on the plot:

- At $t = \tau$ seconds = 0.5s the inductor has charged to $(1 - e^{-1}) = 63\%$ of the $i(\infty)$ value. This is marked with the dashed blue lines.
- When the input pulse switches down, or off, we see that the inductor has discharged to $e^{-1} = 37\%$ of its (interim) ‘initial’ value of 1.729 A, at time $t = 1.5$ s. This is one time constant after the input pulse switches off.

%% Prob5.m
%% Script plots the solution for the RL circuit input pulse response
%% J Cardell, March 2015
%%
%% time vectors for the input pulse, defined for the charging and
%% discharging time periods of the circuit response
t1 = [0:0.01:1];
t2 = [1:0.01:4];

%% Current function for charging and discharging, in response to the input
%% pulse
i1 = 2*(1 - exp(-2*t1));
i2 = 1.729*exp(-2*(t2-1));

%% To see the full 'charging' response, plot i1 for 4 seconds
t1b = [ 0:0.01: 4];
i1b = 2*(1 - exp(-2*t1b));

%% Plot the output of the circuit, i(t)
plot (t1, i1, 'b', t2, i2, 'm',t1b, i1b, 'b:', 'LineWidth', 2)
xlabel ('time (seconds)', 'FontSize',14)
ylabel ('i(t) (amps)', 'FontSize',14)
title ('RL Circuit Response to an Input Pulse', 'FontSize',14)
8) given the following circuit

Racetrack = 4 km

assume \( V_0 = 0 \) V

let \( V_{oo} = 120 \) V

\[ T = RC = 510 \text{ s} \]

so

\[ V(t) = 120 \left( 1 - e^{-t/510} \right) \text{ V} \]

if

\[ V(t) = 85.6 \text{ V} \] then

\[ \frac{85.6}{120} = 1 - e^{-t/510} \]

\[ 0.287 = e^{-t/510} \]

so

\[ t = 6375 \text{ s} \]

speed = \( \frac{4}{6375} = \frac{6.28}{510} \) m/s

As stated, there is no initial stored energy so \( i_n(t) = 0 \)

**Forced Response** = \( 120 \left( 1 - e^{-t/510} \right) \text{ V} \)