CHAPTER 3

Problem 1

• I selected $R_1 = 4 \, k\Omega$, $R_2 = 2 \, k\Omega$, and $R_3 = 2 \, k\Omega$. You could select anything other than all the $R = 1$. Note that finding negative current, or even a 0 A current, is perfectly fine. 0A, for example indicates zero net current is flowing through the given element.

• Determine the value of $I_x$ using nodal analysis.

• Label the node voltage in the top middle of the circuit as $V_x$.

Apply KCL at the top middle node
\[ \frac{(12 - V_x)}{4k} = \frac{(V_x - 0)}{2k} + \frac{(V_x - 9)}{2k} \]

Now, multiply through by 4k and rearrange to find

$(1+2+2) \, V_x = 12+18 = 30$

or $V_x = 30/5 = 6 \, V$

$I_x = 6/(2k) = 3 \, mA$

Problem 2

Step 1. The first thing to do is to select a reference node and to identify all the unknown nodes.

• Select the bottom of the circuit as the reference node.

• The only unknown node is the one connecting all the resistors together in the middle, and we will call that node $V_1$.

• The other two nodes are at the top of each source.

• Relative to the reference, the one at the top of the 10V source is +10 V.

• The one at the top of the 20V source is +20 V.

Step 2. Setup the nodal equation (there is only one since there is only one unknown node), setting the current(s) flowing into the node equal to the current(s) flowing out.

\[
\frac{10 - V_1}{10} = \frac{V_1 - 0}{10} + \frac{V_1 - 20}{4} \quad \Rightarrow \quad \frac{3(10 - V_1)}{10} = \frac{V_1 - 0}{10} + \frac{V_1 - 20}{4}
\]
\[ 60 - 6V_1 = 2V_1 + 5V_1 - 100 \]
\[ V_1 = 12.3V \]

**Step 3.** The answer can be checked by calculating all the currents and see if they obey KCL:

\[ \Sigma I_{in} = \Sigma I_{out} \Rightarrow \frac{20 - 12.3}{4} = \frac{12.3 - 0}{10} + \frac{12.3 - 10}{10/3} \]

\[ 1.92 = 1.23 + 0.69 \leftarrow \text{Yes, KCL is confirmed} \]

**PROBLEM 3** – same circuit as in problem 2, now using mesh analysis

![Circuit Diagram]

- \( I_1 \) drawn clockwise flowing out of the 10V source positive terminal
- \( I_2 \) drawn counter-clockwise, flowing out of the 20V source positive terminal

\[ \Sigma V_{loop} = 0 \Rightarrow -10 + \left( \frac{50}{15} \right) I_1 + 10(I_1 + I_2) = 0 \]
\[ \Rightarrow -20 + 4I_2 + 10(I_2 + I_1) = 0 \]

Solving these equations leads to:

\[ 40I_1 + 30I_2 = 30 \Rightarrow 4I_1 + 3I_2 = 3 \Rightarrow I_1 = -\frac{3}{4}I_2 + \frac{3}{4} \]
\[ 10I_1 + 14I_2 = 20 \Rightarrow 10(-\frac{3}{4}I_2 + \frac{3}{4}) + 14I_2 = 20 \]

Solving for mesh currents:

\[ 6.5I_2 = 12.5 \text{ or } I_2 = 1.92 \text{ A} \]
\[ I_1 = -0.69 \text{ A} \]

\( I_1 \) for this solution was assumed to be **positive** flowing out of the +10V terminal. A solution of -0.69 A thus indicates that this assumption for the direction of current flow was incorrect, and in fact the current is flowing **into** positive terminal of the 10V source.

**Solving for \( V_1 \)**

\[ 10(I_1 + I_2) \Rightarrow 10(-0.69 + 1.92) = 12.3 \text{ V} \]

\( \text{The same answer as above.} \)
Problem 4

For loop 1,
\[ 6 = 12i_1 - 2i_2 \quad \rightarrow \quad 3 = 6i_1 - i_2 \quad (1) \]

For loop 2,
\[ -8 = -2i_1 + 7i_2 - i_3 \quad (2) \]

For loop 3,
\[ -8 + 6 + 6i_3 - i_2 = 0 \quad \rightarrow \quad 2 = -i_2 + 6i_3 \quad (3) \]

We put (1), (2), and (3) in matrix form,

\[
\begin{bmatrix}
6 & -1 & 0 \\
2 & -7 & 1 \\
0 & -1 & 6
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
8 \\
2
\end{bmatrix}
\]

\[ \Delta = \begin{vmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{vmatrix} = -234, \quad \Delta_2 = \begin{vmatrix} 3 & 0 \\ 2 & 8 \end{vmatrix} = 240 \]

\[ \Delta_3 = \begin{vmatrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & -1 & 2 \end{vmatrix} = -38 \]

At node 0, \( i + i_2 = i_3 \) or \( i = i_3 - i_2 = \frac{\Delta_3 - \Delta_2}{\Delta} = \frac{-38 - 240}{-234} = 1.188 \ A \)

Using a Computer: You can use Matlab or Mathematica (if you learned that in PHY210), or any other method, to solve the simultaneous equations.

\[
\begin{bmatrix}
6 & -1 & 0 \\
2 & -7 & 1 \\
0 & -1 & 6
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
8 \\
2
\end{bmatrix}
\]

\[ i = \text{inv}(A) \cdot B \]

\[ i = 0.3291 \]


\[-1.0256
0.1624\]

>> i_answer = i(3) - i(2)

\[i\_answer = 1.1880\]

Alternatively, you can solve these simultaneous equations by hand, through substitution, ultimately finding \( i = 1.188\,A \)

**Problem 5**

Let \( v_3 \) be the voltage between the 2k\( \Omega \) resistor and the voltage-controlled voltage source. At node 1,

\[
3\times\frac{1}{4000} = \frac{v_1 - v_2}{4000} + \frac{v_1 - v_3}{2000} \rightarrow 12 = 3v_1 - v_2 - 2v_3 \quad (1)
\]

At node 2,

\[
\frac{v_1 - v_2}{4} + \frac{v_1 - v_3}{2} = \frac{v_2}{1} \rightarrow 3v_1 - 5v_2 - 2v_3 = 0 \quad (2)
\]

Note that \( v_0 = v_2 \). We now apply KVL in Fig. (b)

\[-v_3 - 3v_2 + v_2 = 0 \quad v_3 = -2v_2 \quad (3)\]

From (1) to (3)

\[v_1 = 1\,V, \quad v_2 = 3\,V\]
Problem 6

KCL: \( i_1 + i_2 + i_3 = 0 \) \( \rightarrow \) Substituting Ohm’s Law \( \rightarrow \) \( \frac{v_1}{10} + \frac{(v_1 - 60) - 0}{20} + \frac{v_1 - 5v_0}{20} = 0 \)

But using the voltage divider rule we know: \( v_0 = \frac{2}{5}v_1 \) so then \( v_1 + v_1 - 60 + v_1 - 2v_1 = 0 \)

or \( v_1 = 60/2 = 30 \) V, therefore \( v_0 = \frac{2}{5}v_1 = 12 \) V

Problem 7

(1) For loop 1, \( i_1 = 5 \) A

(2) For loop 2, writing KVL and substituting in Ohm’s Law as you write KVL: \( -40 + 7i_2 - 2i_1 - 4i_3 = 0 \)

which leads to \( 50 = 7i_2 - 4i_3 \)

Note:
- that the “7” coefficient comes from \( 1\Omega + 2\Omega + 4\Omega = \) the sum of all the resistance encountered by \( i_2 \)
- the opposing directions of \( i_1 \) and \( i_2 \) and of \( i_2 \) and \( i_3 \). This means that they will have opposites signs in the KVL equations.

(3) For loop 3, \( -20 + 12i_3 - 4i_2 = 0 \) which leads to \( 5 = -i_2 + 3i_3 \)

Solving with (2) and (3):

\( i_2 = 10 \) A, \( i_3 = 5 \) A

And finally:
\( v_0 = 4(i_2 - i_3) = 4(10 - 5) = 20 \) V
**Problem 8** → Using superposition

Let \( V_o = V_1 + V_2 \), where \( V_1 \) and \( V_2 \) are due to 9-V and 3-V sources respectively. To find \( V_1 \), consider the circuit below.

\[
\frac{9 - V_1}{3} = \frac{V_1 + V_1}{9} \quad \rightarrow \quad V_1 = \frac{27}{13} = 2.0769
\]

To find \( V_2 \), consider the circuit below.

\[
\frac{V_2}{9} + \frac{V_2}{3} = \frac{3 - V_2}{1} \quad \rightarrow \quad V_2 = \frac{27}{13} = 2.0769
\]

\[ V_o = V_1 + V_2 = 4.1538 \text{ V} \]

**Extra** → Using source transformation

\( V_o \) is the voltage drop across all three branches: (1) 4Ω + 5Ω resistors, (2) 3Ω + 9V source, and (3) 1Ω + 3V source. If we are using source transformation, it will be clearest to maintain the 4Ω + 5Ω = 9Ω branch as where we identify \( V_o \).
Use source transformation to find \( V_0 \)

Transform the middle branch:

\[
\begin{align*}
3\Omega & \quad 9V \\
\downarrow & \quad \uparrow \\
\quad & \quad \|
\end{align*}
\]

\( I_{S1} = \frac{9V}{3\Omega} = 3A \)

Transform rightmost branch:

\[
\begin{align*}
3V & \quad 1\Omega \\
\downarrow & \quad \uparrow \\
\quad & \quad \|
\end{align*}
\]

\( I_{S2} = \frac{3V}{1\Omega} = 3A \) \( \rightarrow \) pure current

That \( I_{S1} = I_{S2} \)

Combining center and rightmost branches leads to:

\[
\begin{align*}
3A & \quad 3A \\
\downarrow & \quad \uparrow \\
\quad & \quad \|
\end{align*}
\]

\( I_{S1} + I_{S2} \Rightarrow 6A \)

\[
\begin{align*}
3\Omega & \quad 1\Omega \quad 3\Omega \quad 1\Omega \\
\downarrow & \quad \uparrow & \quad \| & \quad \| \\
\quad & \quad \|
\end{align*}
\]

\( 0.75\Omega \)

Options for final step:

1. \( 6A \quad 3\Omega \quad 9\Omega \quad 6A \quad 0.89\Omega \quad V_0 = 4.15V \)

\( (\frac{3}{4}||9 = 0.89\Omega) \)

2. Transform one more time, the \( I_5 = 6A \) and \( \frac{3}{4}\Omega \) source:

\[
\begin{align*}
4.5V & \quad 9\Omega \\
\downarrow & \quad \uparrow \\
\quad & \quad \|
\end{align*}
\]

Use voltage divider:

\[ V_0 = 4.5 \left( \frac{9}{9.75} \right) = 4.15V \]

\( (4.5V = (6A)(\frac{3}{4}\Omega)) \)