CHAPTER 3

Problem 1

I selected $R_1 = 4 \, \text{k}\Omega$, $R_2 = 2 \, \text{k}\Omega$, and $R_3 = 2 \, \text{k}\Omega$. You could select anything other than all the $R = 1$. Note that finding negative current, or even a 0 A current, is perfectly fine. 0A, for example indicates zero net current is flowing through the given element.

- Determine the value of $I_x$ using nodal analysis.
- Label the node voltage in the top middle of the circuit as $V_x$.

Apply KCL at the top middle node
\[\frac{(12 - V_x)}{4k} = \frac{(V_x - 0)}{2k} + \frac{(V_x - 9)}{2k}\]

Now, multiply through by 4k and rearrange to find
(1+2+2) $V_x = 12 + 18 = 30$

or $V_x = 30/5 = 6 \, \text{V}$

$I_x = 6/(2k) = 3 \, \text{mA}$

Problem 2

Step 1. The first thing to do is to select a reference node and to identify all the unknown nodes.

- Select the bottom of the circuit as the reference node.
- The only unknown node is the one connecting all the resistors together in the middle, and we will call that node $V_1$.
- The other two nodes are at the top of each source.
- Relative to the reference, the one at the top of the 10V source is +10 V.
- The one at the top of the 20V source is +20 V.

Step 2. Setup the nodal equation (there is only one since there is only one unknown node), setting the current(s) flowing into the node equal to the current(s) flowing out.

\[
\frac{10 - V_1}{10} = \frac{V_1 - 0}{10} + \frac{V_1 - 20}{4} \Rightarrow \frac{3(10 - V_1)}{10} = \frac{V_1 - 0}{10} + \frac{V_1 - 20}{4}
\]
\[ 60 - 6V_1 = 2V_1 + 5V_1 - 100 \]
\[ V_1 = 12.3V \]

**Step 3.** The answer can be checked by calculating all the currents and see if they obey KCL:

\[ \Sigma I_{in} = \Sigma I_{out} \Rightarrow \frac{20 - 12.3}{4} = \frac{12.3 - 0}{10} + \frac{12.3 - 10}{10/3} \]

\[ 1.92 = 1.23 + 0.69 \leftarrow \text{Yes, KCL is confirmed} \]

**PROBLEM 3** – same circuit as in problem 2, now using mesh analysis

\[ \Sigma V_{loop} = 0 \Rightarrow -10 + \left( \frac{50}{15} \right) I_1 + 10(I_1 + I_2) = 0 \]
\[ \Rightarrow -20 + 4I_2 + 10(I_2 + I_1) = 0 \]

Solving these equations leads to:

\[ 40I_1 + 30I_2 = 30 \Rightarrow 4I_1 + 3I_2 = 3 \Rightarrow I_1 = \frac{-1}{4}I_2 + \frac{3}{4} \]
\[ 10I_1 + 14I_2 = 20 \Rightarrow 10(-\frac{1}{4}I_2 + \frac{3}{4}) + 14I_2 = 20 \]

Solving for mesh currents:

\[ 6.5I_2 = 12.5 \quad \text{or} \quad I_2 = 1.92 \ A \]
\[ I_1 = -0.69 \ A \] \( \to \) \( I_1 \) for this solution was assumed to be *positive* flowing out of the +10V terminal. A solution of -0.69 A thus indicates that this assumption for the direction of current flow was incorrect, and in fact the current is flowing *into* positive terminal of the 10V source.

**Solving for \( V_1 \)**

\[ 10(I_1 + I_2) \Rightarrow 10(-0.69 + 1.92) = 12.3 \ A \] \( \to \) The same answer as above.
Problem 4

Using a Computer: You can use Matlab or Mathematica (if you learned that in PHY210), or any other method, to solve the simultaneous equations.

```matlab
>> A = [6 -1 0; -2 7 -1; 0 -1 6]
A =
6 -1 0
-2 7 -1
0 -1 6

>> B = [3; -8; 2]
B =
3
-8
2

>> i = inv(A)*B
i =
0.3291
```
>> i_answer = i(3) - i(2)

\[ i\textunderscore answer = 1.1880 \]

Alternatively, you can solve these simultaneous equations by hand, through substitution, ultimately finding \( i = 1.188\)A.

**Problem 5**

Let \( v_3 \) be the voltage between the 2kΩ resistor and the voltage-controlled voltage source. At node 1,

\[ 3\times10^{-3} = \frac{v_1 - v_2}{4000} + \frac{v_1 - v_3}{2000} \quad \rightarrow \quad 12 = 3v_1 - v_2 - 2v_3 \quad (1) \]

At node 2,

\[ \frac{v_1 - v_2}{4} + \frac{v_1 - v_3}{2} = \frac{v_2}{1} \quad \rightarrow \quad 3v_1 - 5v_2 - 2v_3 = 0 \quad (2) \]

Note that \( v_0 = v_2 \). We now apply KVL in Fig. (b)

\[ -v_3 - 3v_2 + v_2 = 0 \quad v_3 = -2v_2 \quad (3) \]

From (1) to (3)

\[ v_1 = 1 \text{ V}, \quad v_2 = 3 \text{ V} \]
Problem 6

![Image of circuit diagram]

KCL: \( i_1 + i_2 + i_3 = 0 \) \( \rightarrow \) Substituting Ohm’s Law \( \rightarrow \)

\[
\frac{v_1}{10} + \frac{(v_1 - 60) - 0}{20} + \frac{v_1 - 5v_0}{20} = 0
\]

But using the voltage divider rule we know: \( v_o = \frac{2}{5}v_1 \) so then \( v_1 + v_1 - 60 + v_1 - 2v_1 = 0 \)

or \( v_1 = 60/2 = 30 \text{ V} \), therefore \( v_0 = \frac{2}{5}v_1 = 12 \text{ V} \)

Problem 7

(1) For loop 1, \( i_1 = 5 \text{ A} \)

(2) For loop 2, writing KVL and substituting in Ohm’s Law as you write KVL: 

\(-40 + 7i_2 - 2i_1 - 4i_3 = 0\)

which leads to \( 50 = 7i_2 - 4i_3 \)

Note:
- that the “7” coefficient comes from \( 1\Omega + 2\Omega + 4\Omega = \) the sum of all the resistance encountered by \( i_2 \)
- the opposing directions of \( i_1 \) and \( i_2 \) and of \( i_2 \) and \( i_3 \). This means that they will have opposites signs in the KVL equations.

(3) For loop 3, 

\(-20 + 12i_3 - 4i_2 = 0\)

which leads to \( 5 = -i_2 + 3i_3 \)

Solving with (2) and (3):

\( i_2 = 10 \text{ A}, i_3 = 5 \text{ A} \)

And finally:

\( v_0 = 4(i_2 - i_3) = 4(10 - 5) = 20 \text{ V} \)
**Problem 8** → Using superposition

Let \( V_o = V_1 + V_2 \), where \( V_1 \) and \( V_2 \) are due to 9-V and 3-V sources respectively. To find \( V_1 \), consider the circuit below.

\[
\frac{9 - V_1}{3} = \frac{V_1}{9} + \frac{V_1}{1} \quad \rightarrow \quad V_1 = 27/13 = 2.0769
\]

To find \( V_2 \), consider the circuit below.

\[
\frac{V_2}{9} + \frac{V_2}{3} = \frac{3 - V_2}{1} \quad \rightarrow \quad V_2 = 27/13 = 2.0769
\]

\[ V_o = V_1 + V_2 = 4.1538 \text{ V} \]

**Problem 9 on own, for practice** → Using source transformation

\( V_o \) is the voltage drop across all three branches: (1) 4Ω + 5Ω resistors, (2) 3Ω + 9V source, and (3) 1Ω + 3V source. If we are using source transformation, it will be clearest to maintain the 4Ω + 5Ω = 9Ω branch as where we identify \( V_o \).
Use source transformation to find $V_0$

Transform the middle branch

$3\Omega$ $\oplus$ $9\text{V}$ $\rightarrow$ $I_{s1}$ $\uparrow$ $3\Omega$ $\rightarrow$ $I_{s1} = \frac{9\text{V}}{3\Omega} = 3\text{A}$

Transform rightmost branch

$3\text{V}$ $\oplus$ $I_{s2}$ $\rightarrow$ $I_{s2} = \frac{3\text{V}}{1\Omega} = 3\text{A} \Rightarrow$ pure coincidence that $I_{s1} = I_{s2}$

Combining center and rightmost branches leads to

$I_{s1} + I_{s2} \Rightarrow 6\text{A}$

Options for final step

1. $6\text{A} \oplus \frac{3\text{V}}{4\Omega} \ll 9\Omega = 6\text{A} \uparrow$ $0.69\Omega \Rightarrow V_0 = 4.15\text{V}$

2. Transform once more time, the $I_S = 6\text{A}$ and $\frac{3}{4} \text{V} - \text{R}$ source:

$V_0 = 4.5\text{V} \left(\frac{9}{9.75}\right) = 4.15\text{V}$

$(4.5\text{V} = (6\text{A})(\frac{3}{4}\Omega))$