Problem 1

\[ V_0 = V_s \left( \frac{R||Z_C}{Z_L + R||Z_C} \right) \]

Find \( R||Z_C = \frac{R/SC}{R + \frac{1}{2}SC} = \frac{R}{1+sRC} \)

So \( \frac{V_0}{V_s} = \frac{R/(1+sRC)}{sL + \frac{R}{1+sRC}} \)

\[ = \frac{R}{s^2RC + sL + R} = \frac{N(s)}{D(s)} = H(s) \]

b & c) Terms in the numerator, \( N(s) \), with "s" represent zeros, and terms in the denominator, \( D(s) \), with "s" represent poles.

The numerator, \( N(s) = R \Rightarrow \) there are no terms with "s" so there are no zeros for this transfer function.

The denominator, \( D(s) \), is a polynomial that can be factored into \( (s+p_1)(s+p_2) \). Since this is a second order polynomial with two roots, there are two poles for this transfer function.

Extra: Checking type of filter we find

\[ |H(s)| \big|_{w=0} = \frac{R}{R} = 1 \]

\[ |H(s)| \big|_{w=\infty} = \frac{R}{\infty} = 0 \]

So this is a low pass filter.
**Problem 2**

\[ \frac{V_o}{V_s} = \frac{R \parallel Z_L}{Z_C + R \parallel Z_L} = \frac{sRL/(R+SL)}{\frac{1}{SC} + \frac{sRL}{R+SL}} = \frac{sRL}{(R+SL) \left( \frac{R+SL}{SC} + sRL \right)} \]

\[ = \frac{s^2RLC}{s^2RLC + SL + R} = \frac{N_o(s)}{D(s)} = H_o(s) \]

(a) Find the transfer function

(b) Zeros: a double zero at the origin

(c) Poles: As for problem 1, D(s) is a second order polynomial with two roots, so there are two poles

**Extra:** Checking filter type

@ \( w = 0 \) \( |H(s)| = 0 \)

@ \( w = \infty \) \( |H(s)| \to \frac{\infty}{\infty} \to 1 \)

so this is a high pass filter
PROBLEM 3

\[ R_s = 12k\, \Omega \]

\[ R = 45k\, \Omega \]

\[ C = 1\, \mu F \]

\[ L = 60mH \]

Find the transfer function \( H(s) = \frac{V_o(s)}{V_s(s)} \)

let \( Z_0 = R \parallel Z_C \parallel Z_L \) then \( H(s) = \frac{Z_0}{R_s + Z_0} \)

Find \( Z_0 : \frac{Z_C}{Z_L} = \frac{\frac{SL}{SC}}{\frac{1}{SC}} = \frac{SL}{S^2LC + 1} \)

\[ R \parallel (Z_C || Z_L) = \frac{SRL}{R + \frac{SL}{S^2LC + 1}} = \frac{SRL}{S^2RLC + SL + R} \]

Then \( H(s) = \frac{SRL/(S^2RLC + SL + R)}{R_s + [SRL/(S^2RLC + SL + R)]} = \frac{SRL}{S^2R_sRLC + S(R_s + R)L + R_sR} \)

Substitute element values into \( H(s) \) to find

\[ H(s) = \frac{SRL}{S^2(32.4) + S(3420) + 540M} \]

using Matlab: \( \text{roots}([32.4 3420 540 55]) \)

we find \( p_{1,2} = -52.5 \pm j4082 \)

so \( H(s) = \frac{2700S}{(S + p_1)(S + p_2)} \)
Looking in chapter 14 and reading about resonance, we see that in order for a system to oscillate— for any of the output behavior to be oscillatory— there must be at least one pair of poles (roots) that are a complex conjugate pair (or a pair of zeros that are a complex conjugate pair).

Think about the mathematics of sine & cosine waves, Euler's formula and complex numbers of the form $r \cos \theta + j r \sin \theta$. They all are used to represent mathematically what we observe as oscillatory behavior in the physical world.

For now, we can identify the real part of the poles as "d" the damping coefficient, and the complex term as the frequency of oscillation, as we did for second order circuits.

\[ \text{pole} = -\alpha \pm j \omega_d \]
PROBLEM 3 cont p3

Returning to the transfer function

\[ H(s) = \frac{2700s}{(s+p_1)(s+p_2)} \]

with \( p_{1,2} = -52.5 \pm j4082 \)

we find: zero at the origin

complex pole \( @ \ 4082 \text{ rad/s} \)

OR for now it is fine to say

\[ p_1 = -52.5 + j4082 \]
\[ p_2 = -52.5 - j4082 \]

**Resonant frequency**

\[ \omega_0 = \frac{1}{\text{VLC}} = 4082 \text{ (hmm, looks familiar) rad/s} \]

so \( f_0 = 650 \text{ Hz} \)

**Filter?** (see also Matlab output)

frequencies to check are \( \omega = 0 \), \( \omega = \infty \) and the zeros +/or poles as needed

\[ |H(s)|_{\omega=0} = 0 \quad \text{and} \quad |H(s)|_{\omega=\infty} = \frac{\infty}{\infty} \rightarrow 0 \]

so check \( \omega = \omega_0 \) and we find \( |H(s)| \rightarrow \infty \)

This is a band pass filter (resonant filter)
Problem 3 Solution with Matlab:

Matlab assistance for this problem results in:

\[
\text{>> HS = tf([2700 0], [32.4 3420 540E6])}
\]

\[
\begin{align*}
\text{2700 s} \\
\text{--------------------------} \\
\text{32.4 s^2 + 3420 s + 5.4e08}
\end{align*}
\]

\[
\text{>> roots([32.4 3420 540E6])}
\]

\[
1.0e+03 * [-0.0528 + 4.0821i \\
-0.0528 - 4.0821i]
\]

\[
\text{>> bode(HS)}
\]

We can see from this plot that the filter is a resonant circuit band-pass filter with a sharp peak in the amplitude of the transfer function at the resonant frequency.
**Problem 4 Solution with Matlab** (hand drawn Bode plot after Matlab solution):

For the circuit below

a) Find the voltage gain transfer function
   a. Identify the frequencies that contribute a zero to the transfer function
   b. Identify the frequencies that contribute a pole to the transfer function
   c. Identify the gain of the transfer function

b) Construct the Bode plot (amplitude and phase)
   a. Label all important points and elements on the Bode plot
   b. Useful graph paper is linked below the link for the Lab 10 handout
   c. Identify what type of filter this circuit represents (high pass? low pass? band pass? notch? other?)

As with problem #1, the transfer function is: \( \frac{R}{s^2RLC+slR} \).

If the element values are substituted into this expression, we find

\[
H(s) = \left( \frac{\frac{1}{4}}{s^2 + 4s + \frac{1}{4}} \right) = \left( \frac{1}{s^2 + 4s + 1} \right) = \left( \frac{1}{(s+3.7)(s+0.3)} \right) = \left( \frac{1}{(1+s/3.7)(1+s/0.3)} \right)
\]

**Zeros and Poles:**

This function has 3 elements (there are no zeros)

a) A gain of 1 (unity) \( \Rightarrow \) Note that if you factor the denominator, \( D(s) \), and do not round the values of the poles (as I did in typing ‘3.7’ and ‘0.3’), then the product of \( (1/p_1) \times (1/p_2) \) (the value of the gain once you put \( H(s) \) into standard form) does equal 1. \( \Rightarrow K = (1/p_1) \times (1/p_2) = 1 \)

b) A pole at 3.7

c) A pole at 0.3

Each of these elements plotted individually and then in aggregate is shown below:
a) Bode plot for a gain of 1 (unity) – this contributes nothing to the final plot

b) Bode plot for a pole at 0.3 – slope of -20dB/decade starting at 0.3rad/s and a phase shift from 0 to -90 degrees between 0.03 and 3 rad/s.
c) Bode plot for a pole at 3.7 – slope of -20dB/decade starting at 3.7rad/s and a phase shift from 0 to -90 degrees between 0.37 and 37 rad/s.
d) Aggregate Bode Plot

For the amplitude plot: Note that for this final plot, the y-axis scale is such that the amplitude response between 0.3 and 3.7 rad/s decreases at a slope of 20dB/decade. (This is the green dashed line.)

The second pole (the purple dashed line) starts acting at 3.7 rad/s.

Starting at 3.7 rad/s the slope is therefore -40dB/decade, since from this frequency and above both poles are acting to block higher frequency energy (the red dashed line).

For the phase plot, the 0.3 rad/s pole contributes a phase shift moving from 0 to -90 degrees starting at 0.03 rad/s and ending at 3 rad/s (starting one decade below the pole and ending one decade beyond the pole).

The second pole, at 3.7 rad/s contributes a second phase shift starting at 0 degrees and ending at -90 degrees. This second phase shifting starts at 0.37 rad/s and ends reaches its -90 degree contribution at 37 rad/s.

Together these poles result in a -180 degree phase shift.
Type of Filter for Problem 4:

Test $\omega = 0 \rightarrow |H(s)| = \frac{1}{(1+0)(1+0)} = 1$

$\omega = \infty \rightarrow |H(s)| = \frac{1}{(1+\infty)(1+\infty)} = 0$

So, this is a low pass filter.
$$H(s) = \frac{1}{s^2 + 4s + 1} = \frac{1}{(s + 3.7)(s + 0.3)} = \frac{1}{(1 + \frac{s}{3.7})(1 + \frac{s}{0.3})}$$

Magnitude (amplitude)

- Pole at $w = 3.7$
- Slope = $-20$ dB/decade

Phase

- Pole at $w = 0.3$
- Slope = $+20$ dB/decade

Final plot has slope of $-40$ dB/decade after 2nd pole at $w = 3.7$

PROBLEM 5

Analyze and plot $H(s) = \frac{250(s+1)}{s(s^2 + 10s + 25)} = \frac{250(s+1)}{s(s+5)^2} \left(\frac{\frac{1}{5}}{\frac{1}{5}}\right)$

$H(s) = \frac{(250/25)(1+5/1)}{s (1+5/5)^2}$

Gain $K = \frac{250}{25} = 10 \rightarrow 20\text{dB}$

So gain $K=10 \Rightarrow$ Note that you must put $H(s)$ into standard form in order to determine the correct value for the gain.

Zero @ $w=1$.

Pole @ the origin.

Double pole @ $w=5$.

Check for type of filter behavior.

@ $w=0$ \( |H(s)| = \frac{10}{0} \rightarrow \text{very large, so this circuit passes low frequency input through the circuit to the output} \)

@ $w=\infty$ \( |H(s)| = \frac{\infty}{\infty^3} \rightarrow 0 \) so this circuit blocks high frequency input signals from passing through to the output.

Therefore this is a low pass filter.
EGR 220 Chapter 14 Homework

\[ H(s) = \frac{10(1+s)}{s(1+\frac{s}{5})^2} \]

**Magnitude (amplitude)**

- Zero at \( w=1 \)
- Gain of 10 (20dB)
- Pole at origin
- Double pole @ \( w=5 \)

**Phase**

- Zero @ \( w=1 \)
- Pole @ origin
- Double pole @ \( w=5 \)