Old 220 Exam 1 questions

Briefly define/explain NODAL analysis. (Be clear, concise and complete.)

refer to Kirchhoff's laws, what equations are used (i.e. KCL) and what is being solved for (i.e. node voltages). And Ohm's law is used also ... (where?) See Text

Briefly define/explain MESH analysis. (Be clear, concise and complete.)

See above answer, + adapt for mesh analysis

Power and energy:

- With words or math, state the relationship between power and energy.
- With words or math, state the relationship between power, voltage and current.
- State the standard units for:
  - Energy
  - Power
  - Voltage
  - Current

See text!

For the circuit below, an equivalent circuit would consist of:

a) A 20 V source in series with a 100Ω resistor
b) A 20 A source in parallel with a 5Ω resistor
c) A 4 A source in parallel with a 5Ω resistor
d) A 4 V source in series with a 100Ω resistor
e) None of the above

![Circuit diagram]
For the circuit below, an equivalent circuit would consist of:

a) A 20 V source in series with a 100Ω resistor
b) A 20 A source in parallel with a 5Ω resistor
c) A 4 A source in parallel with a 5Ω resistor
d) A 4 V source in series with a 100Ω resistor
e) None of the above

For the circuit below, an equivalent circuit would consist of:

a) A 20 V source in series with a 100Ω resistor
b) A 20 A source in parallel with a 5Ω resistor
c) A 4 A source in parallel with a 5Ω resistor
d) A 4 V source in series with a 100Ω resistor
e) None of the above

For 3 parallel resistors of different resistance, $R_1 > R_2 > R_3$, the equivalent resistance is

a) Greater than the value of $R_3$
b) Less than the value of $R_1$
c) Less than the value of $R_3$
d) Greater than the value of $R_1$
e) None of the above
The current supplied by the source for the circuit below is

a) \( I = 1.25 \, \text{A} \)

b) \( I = 2 \, \text{A} \)

c) \( I = 3.3 \, \text{A} \)

d) None of the above

**DERIVE** the expression for the equivalent resistance for the circuit below.

- Start with Ohm's Law, Kirchhoff's Current Law (KCL), and Kirchhoff's Voltage Law (KVL)
  - You may use the current as labeled: \( I_t \) for the 'total' current into the circuit at node \( a \), \( I_c \) for the current through the branch with \( R_1 \) and \( R_2 \), and \( I_d \) for the current through the branch with \( R_3 \)
  - You may not use expressions for series or parallel equivalent resistance without first deriving them.
- End with an expression for \( R_{eq} = f[R_1, R_2, R_3] \) (i.e., \( R_{eq} \) is equal to an expression containing only \( R_1, R_2, R_3 \))

See §2.5 + 2.6 for developing \( R_{eq} \) expressions by starting with Ohm's law and using KVL or KCL as needed.
Label the meshes on the circuit below and write a set of mesh equations in matrix format that could be used to solve for the mesh currents. (Do not solve for these currents, simply write a set of matrix equations.)
(Note for 2013: I will not give you a circuit with an inductor, as we have not seen these yet)

\[ -24 + 12I_1 + 60(I_1 - I_2) = 0 \]
\[ 60(I_2 - I_1) + 6(I_2 - I_3) + 3(I_2 - I_3) = 0 \]
\[ 3(I_3 - I_2) + 6(I_3 - I_2) + 40(I_3) = 0 \]

\[ 72I_1 - 60I_2 + 0 = 24 \]
\[ -60I_1 + 69I_2 - 9I_3 = 0 \]
\[ 0 - 9I_2 + 49I_3 = 0 \]

\[
\begin{pmatrix}
72 & -60 & 0 \\
-60 & 69 & -9 \\
0 & -9 & 49
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2 \\
I_3
\end{pmatrix}
= 
\begin{pmatrix}
24 \\
0 \\
0
\end{pmatrix}
\]
a) Use nodal analysis to find the currents I₁ and I₂
b) Use superposition to find the currents I₁ and I₂

Nodal

A \( V_1 = V_2 \) so have **ONE** equation
also have **ONE** unknown, \( V_1 \) (since \( V_1 + V_2 \) are the same)

KCL: \( I_{S1} + 2 = I_1 + I_2 \)

\[
\frac{10 - V_1}{5} + 2 = \frac{V_1 - 0}{10} + \frac{V_1 - 0}{2} \Rightarrow 20 - 2V_1 + 20 = V_1 + 2V_1
\]

\( 40 = 5V_1 \) or \( V_1 = 8 \)

using Ohm's law \( I_1 = \frac{8}{10} \approx 0.8 \text{ A} \); \( I_2 = \frac{8}{5} = 1.6 \text{ A} \)

**IF** there were a resistor between \( V_1 + V_2 \) then
these would **NOT** be the same node. You would
then have **2** equations & **2** unknowns
a) Use nodal analysis to find the currents $I_1$ and $I_2$

b) Use superposition to find the currents $I_1$ and $I_2$

\[\begin{align*}
5\Omega & \quad V_1 & \quad 2\Omega \\
10V & \quad 10\Omega & \quad 2A & \quad 3\Omega \\
I_3 & \quad I_1 & \quad I_2
\end{align*}\]

First set $I_5 = 0$

- open circuit that branch

Notice $10\Omega$ and $(2+3)\Omega$ branches are in parallel, since they share the same nodes, 0 and 0.

Step 1 - find $I_{\text{total}}$ from 10V source
Step 2 - use current divider to find $I_1 + I_2$

\[\begin{align*}
(I) & \quad I_5 = \frac{10V}{5+(10/5)} = \frac{10}{5+10/5} = 1.2A \\
(II) & \quad I_1 = 1.2 \frac{5}{10+5} = 0.4A \quad \text{and} \quad I_2 = 1.2 \frac{10}{5+10} = 0.8A
\end{align*}\]

Second set $V_5 = 0$ → short circuit branch

Observe that with $V_5$ shorted, all resistor branches are in parallel.

Using nodal analysis with this single source

\[\begin{align*}
& \Rightarrow 20 = 5V\text{, or } V_1 = 4V \\
\therefore & \quad I_1 = \frac{4}{10} = 0.4A \quad \text{and} \quad I_2 = \frac{4}{5} = 0.8A
\end{align*}\]

CINCEIDENTALLY, both sources contribute equally to current flows

ADD results together

\[\begin{align*}
I_1 \text{ total} & = 0.4 + 0.4 = 0.8A \quad \text{and} \quad I_2 \text{ total} = 0.8 + 0.8 = 1.6A
\end{align*}\]
a) Using Thevenin’s theorem, find \( V_0 \) across the 10\( \Omega \) resistor.

b) Assuming you can change the value of \( R_L \), what value should this load resistor have for maximum power transfer to \( R_L \)? What is this maximum power that could be transferred to the new \( R_L \)?

Step 1 for applying Thevenin’s theorem is to **REMOVE THE LOAD**

Find \( R_{th} \)

Set sources = 0 so \( I_s = \) open clkt and \( V_s = \) short clkt

with \( I_s \) open, \( 4\Omega + 16\Omega \) are in series (w/ \( I_s \) active they ARE NOT in series)

So \( R_L = 1 \Omega + 5\Omega \parallel (4\Omega + 16\Omega) = 1\Omega + 4\Omega = 5\Omega \)

Find \( V_{th} \), with \( R_L \) removed there is no current thru 12R

\( I_1 = 3\text{A} \)

for loop 2: \( 16(I_2 - 3) + 4I_2 + 5(I_2 + 0) + 12 = 0 \)

\( 25I_2 = 36 \)  \( I_2 = 1.44\text{A} \)

\( V_{th} \) is the voltage to ground = 12 + 5(I_2) = 19.2V

a) Our Thevenin equivalent clkt is \( 19.2\pm \frac{5\Omega}{5\Omega} \)

with \( R_L \) reconnected:

\( 19.2V \pm \frac{5\Omega}{5\Omega} \) \( V_o = 19.2 \left( \frac{10}{5+10} \right) = 12.8\text{V} \)
Using Thevenin's theorem, find $V_o$ in the circuit below.

\[ \begin{array}{c}
3 \text{ A} \\
\downarrow \\
16 \Omega \\
\downarrow \\
4 \Omega + \boxed{1 \Omega} \\
\downarrow \\
5 \Omega \\
\downarrow \\
10 \Omega \quad + \\
\downarrow \\
12 \text{ V} \\
\downarrow \\
\boxed{12 \text{ V}} \\
\end{array} \]

\[ \text{part b} \]

For maximum power transfer $R_L = R_{TH}$

\[ R_L = 5 \Omega \]

Now $V_o = \frac{V_{TH}}{2}$

\[ P = \frac{V_o^2}{R_L} = \frac{(\frac{V_{TH}}{2})^2}{R_L} = \frac{19.2^2}{4(5)} \approx 18.4 \text{ W} \]
Find $v_1$ and $v_2$, and the power supplied by each independent source. \(\text{Hint: Think carefully about the simple relationship between the 12V source, } v_o \text{ and } v_i.\)

Use nodal analysis

\[ \text{KCL } v_1: \quad \frac{12 - v_1}{2} = \frac{v_1 - v_2}{8} + 3 + \frac{v_1 - 0}{4} \Rightarrow 48 - 4v_1 = v_1 - v_2 + 24 + 2v_1 \]

\[ \text{KCL } v_2: \quad \frac{v_1 - v_2}{8} + 3 = \frac{v_2 - (5v_0)}{1} \Rightarrow v_1 - v_2 + 24 = 8(v_2 + 5(12 - v_1)) \]

\[ v_1 - v_2 + 24 = 8v_2 + 480 - 40v_1 \]

1. \[ 24 = 7v_1 - v_2 \Rightarrow v_2 = 7v_1 - 24 \]
2. \[ 456 = 41v_1 - 9v_2 \]

\[ 456 = 41v_1 - 9(7v_1 - 24) \]
\[ 240 = -22v_1 \]

\[ v_1 = -10.9 \text{ V} \]
\[ v_2 = -100.4 \text{ V} \]

**Power**

12V source

\[ I_S = \frac{12 - v_1}{2} = 11.45 \]

\[ P = VI = 137.4 \text{ W} \]

5A source

\[ V_{5A} = V_{8.5} = v_1 - v_2 = 89.5 \text{ V} \]

\[ P = VI = 268.5 \text{ W} \]
• **Derive the simple equation for the current divider rule**, that is used to determine how the current from a source is divided as it flows through parallel resistors in a circuit.
  - You want an answer in the form of $i = i(...)$ where $i$ is the current through a resistor in the circuit, $i_s$ is the source current, and the parentheses contain an expression using the resistors in the circuit.
  - State all your assumptions and show all your work.

![Circuit Diagram]

Note that simply writing down the current divider expression will receive 0 points.

Write expressions w/ KVL, KCL & substitute Ohm's law:

**KCL:** $I_s = I_1 + I_2$

**KVL:** $V_s = V_1 = V_2$ and $V_s = I_s R_{eq}$, $V_1 = I_1 R_1$, $V_2 = I_2 R_2$

1. **Find $R_{eq}$ from KCL:** $\frac{V_s}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$ but KVL: $V_s = V_1 = V_2$
   - $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$

2. **Find $I_1$ as a function of $I_s$ and resistors**

   **KVL:** $V_s = V_1$, so $I_s R_{eq} = I_1 R_1 \Rightarrow I_s \left(\frac{R_1 R_2}{R_1 + R_2}\right) = I_1 R_1$

   Finally $I_1 = I_s \left(\frac{R_2}{R_1 + R_2}\right)$

   Likewise $I_2 = I_s \left(\frac{R_1}{R_1 + R_2}\right)$
15) \( i_o \) (the current at the output of the op amp) is equal to
   
   a) Greater than 0A
   b) Equal to 0A
   c) Cannot determine from information given
   d) None of the above

   **Golden Rules**
   
   \( V_+ = V_- \) but \( V_+ = 3V \) so \( V_- = 3V \)
   
   \( I_{in} = 0 \) so \( I_- = 0 \)

   Find \( V_o \):
   
   \( V_o + 1V = V_- = 3V \)
   
   so \( V_o = 2V \)

   KCL:
   
   \( i_o = i_- + i_x = 0 + i_x \) so what is \( i_x \)?
   
   \( i_x = \frac{V_o}{2k} = \frac{2V}{2k} > 0 \)

16) State the two “Golden Rules” used for analyzing ideal op amp circuits

   \[ V_1 \text{ or } V_2 \]

   \( I_{in} \)

   \[ V_o \text{ or } V_+ \]

   \( I_{in} \)

   a) \( V_1 = V_2 \)
   
   b) \( I_{in} = 0 \)
   
   \( V_- = V_+ \)
PART 2
Problem 1: Op Amp (22 points)

a) Find $i_x$, $i_o$, and $v_o$ for the op amp circuit below.
b) Make one or two brief statements about why the values you found make sense.
   (e.g., perhaps referring to Ohm’s law, KVL and/or KCL...)

b) My values make sense because:

Op Amp Circuit:

using Golden rules
i) $4V = V_x = V_-$
   also note $V_a = V_-$
   thus $V_a = 4V$
ii) $i_m = 0 \Rightarrow i_1 = i_2$

Ohm’s Law
$i_x = \frac{V_a - V_0}{2k} = \frac{4}{2k} = 2mA = i_1$

From $V_x$ to ground thru left branch $(3k+2k)\Omega$
$V_x = i_1 \cdot 10A = 2mA \cdot 5k = 10V$

Voltage divider for $V_o$: $V_o = V_x \cdot \frac{3k}{3k+7k} = 10 \cdot \frac{3}{10} = 3V$

Ohm’s Law for $i_x$: $i_x = \frac{V}{R} = \frac{3}{5k} \cdot 1mA$

KCL for $i_o$: $i_o = i_1 + i_x = 2mA + 1mA = 3mA$