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SYMMETRY: AN INTERACTIVE GRAPHICS COMPUTER PROGRAM TO TEACH SYMMETRY RECOGNITION

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Abstract—SYMMETRY is a computer program that will draw two-dimensional motifs and patterns on the screen of a cathode-ray-tube (CRT) terminal. The program enables users to test for the presence of symmetry elements in these patterns by superimposing rotated, reflected, or glide-reflected patterns on the original.

Key Words: Cathode-ray tube, Graphics, FORTRAN, Time sharing, Crystallography, Mineralogy, Symmetry.

INTRODUCTION

This article describes a FORTRAN language interactive computer program written to ease the burden of those who would learn or teach symmetry recognition. The program is designed to enable students to perform conclusive tests for the presence of symmetry elements in two-dimensional objects and patterns. The program relies upon a cathode-ray-tube (CRT) terminal to a time-sharing computer (or minicomputer) with graphics software (CALCOMP or equivalent). The program (1) guides the user through the construction of a two-dimensional pattern using a symmetrical motif, (2) allows the user to examine relationships of possible unit cells to the resulting pattern, and (3) enables the user to test for the presence of symmetry elements at specific locations in the pattern.

Learning to recognize symmetry in objects and patterns can be a surprisingly difficult task for even the best of students. Typically, congruent symmetry elements are identified more easily than enantiomorphous symmetry elements, for congruent symmetry operations may be physically performed on models and patterns. Enantiomorphous symmetry operations, however, cannot be physically performed on models (although real mirrors can be useful). The student must develop an intuitive ability to recognize enantiomorphous symmetry elements and ultimately, of course, congruent symmetry elements as well. The program described in this paper is designed to aid students develop the required intuition by confirming or disproving the presence of symmetry elements proposed by the student.

Verification of the presence of symmetry elements is simple for a computer-drawn pattern. For any given symmetry element, the computer simply performs, mathematically, the appropriate symmetry operation and superimposes the result on the original pattern. If the pattern does indeed possess the symmetry element in question, then the resultant pattern will coincide exactly with the original pattern. If the symmetry element is not present, then an obvious misalignment of the two patterns will exist. Rotation axes, mirror planes, and glide planes all yield to computer inspection, for patterns in the memory of a computer, unlike physical models or drawings, are susceptible to enantiomorphous transformations.

An additional service rendered by the program is to provide an opportunity for students to work with a crystallographic coordinate system. The program user must specify the location in the pattern of the symmetry element to be tested. The only convenient manner to do so is in terms of a coordinate system based on the lattice of the pattern.

Although all symmetry operations of this program are performed in two dimensions, a sense of symmetry developed for two dimensions seems to extend readily to three. Nearly all important symmetry concepts can be introduced using two-dimensional patterns. In addition, two-dimensional patterns are well suited to blackboard presentations, exercises, and exams. I have used the program successfully in an introductory mineralogy course. The advantages outlined here, combined with the additional study incentive provided by the novelty of the computer, make the program a successful teaching aid.

PROGRAM DESCRIPTION

The program is divided into four parts, each of which builds on the preceding part. The option is provided in the program to go back and rerun one or more parts, but it is not possible to skip ahead in the sequence. Each part of the program begins with the computer printing explanatory material and asking the user to input specific instructions. The program then performs appropriate calculations and outputs the results in the graphics mode. The user may study the CRT output while the program waits for the user's signal to proceed.

Part 1 of the program allows the user to select a symmetrical motif which is subsequently (Part 2) repeated by translations to make a two-dimensional pattern. The user may choose one of twelve symbols as a basis for the motif (Fig. 1). The user creates a point-group motif from the chosen symbol by repeating it with a rotation axis and/or a vertical mirror. The option also is available to create a motif with a built in glide plane. A sample of the graphics output from Part 1 is shown as Figure 2.

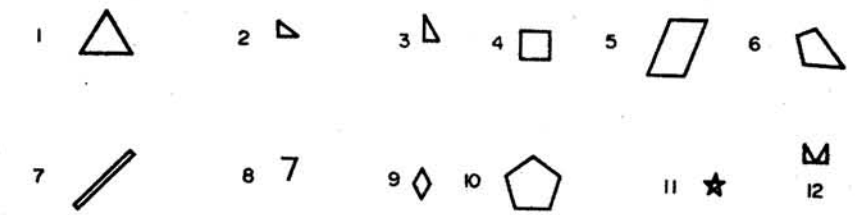

Figure 1. SYMMETRY uses 12 symbols to construct patterns. These symbols are an arbitrary selection of closed figures that can be drawn by connecting a set of five or fewer points. Other figures may be substituted by appropriate modification of DATA statement.

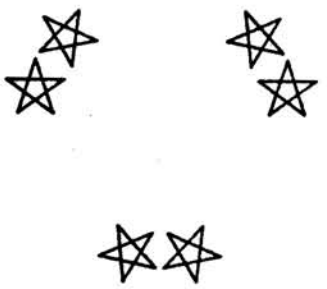

Figure 2. Motif created by repeating a star with 3-fold rotation axis and vertical mirror. Rotation axis is always at origin of coordinate system used to specify symbol in DATA statement.

Part 2 of the program draws a pattern based on the motif of Part 1 according to the instructions of the user. The pattern is constructed using a horizontal translation (T1) and a second translation (T2) selected by the user. The user specifies the angle (A1) between translations T1 and T2 and also the ratio of the lengths of the two translations. The absolute length of T2 is fixed by the program relative to the size of the motif. The user specifies the dimensions of the pattern in terms of the numbers of translations T1 and T2 to be shown. A sample of the output of Part 2 is shown as Figure 3.

Part 3 of the program superimposes a set of unit cells on the pattern created in Part 2. The unit cells shown may be either a "standard" set or another set prescribed by the user. The "standard" unit cell has sides defined by the translations T1 and T2 and an origin at the rotation axis of the symmetrical motif. A "standard" unit cell is necessary to define a "standard coordinate system" which is used in Part 4. The user-defined unit cell must have sides which are a linear combination (vector addition) of the translations T1 and T2. An origin for the user-defined unit cell must be specified in terms of the "standard coordinate system". A sample of the output from Part 3 is shown as Figure 4.

Part 4 is the symmetry verification part of the program. Here the user may test the pattern created in Part 2 for the presence of rotation axes, mirror planes, and

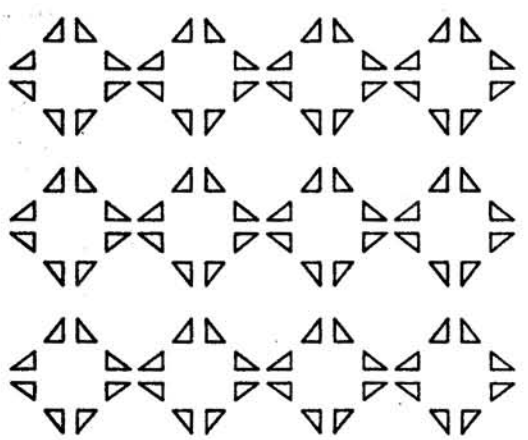

Figure 3. Pattern constructed by Part 2 of program (N1 = 4, N2 = 3). Ratio of T2 to T1 is 1.20 and angle between T1 and T2 is 90 degrees. Motif used comes equipped with 4-fold rotation axis and vertical mirror.

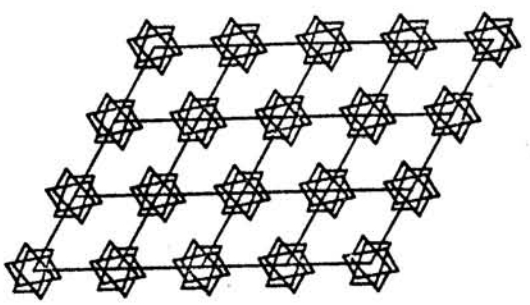

Figure 4. "Standard" unit cell for another computer-drawn pattern. Note that origin of unit cell is rotation axis of motif.

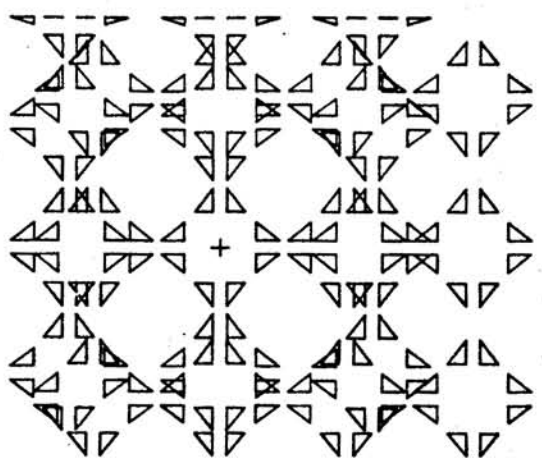

Figure 5. Example of symmetry verification procedure as applied to pattern of Figure 3. Computer tested 4-fold rotation axis identified by cross at coordinates (1,1) of "standard coordinate system". As is evident from obvious overlap, pattern of Figure 3 does *not* possess 4-fold rotation axis at specified location.

glide planes at specific locations. To test for a rotation axis, the user must identify the "fold" of the rotation axis and its location in terms of the standard coordinate system. To test for a mirror plane or glide plane, the user must specify the intercepts of the plane in question on the axes of the standard coordinate system. To test for a glide plane the user must also specify the translational component of the glide in terms of the translations T1 and T2 (by vector addition). In all situations the origin of the standard coordinate system is the center of the lower left motif of the pattern.

The CRT output for Part 4 begins with a redrawing of the original pattern. The computer indicates (with a cross, solid line, or dashed line) the location of the symmetry element to be tested (rotation axis, mirror plane, or glide plane, respectively). The computer draws the pattern again in the appropriate rotated, reflected, or glide-reflected form and orientation. The presence of the tested symmetry element is verified if the two versions of the pattern are coincident.

The program (Appendix) runs on the Control Data Corporation CYBER 70 Series computer at the University of Massachusetts. The CRT terminal used was a TEKTRONIX 4006-1. The scaling routines in the program adjust the graphics output to fit on the TEKTRONIX 8-in by 6-in screen. CALCOMP plotting software is used, slightly modified to suit the TEKTRONIX CRT terminal. Permanent copies of the CRT displayed patterns, shown as figures here, were obtained using a TEKTRONIX Hard Copy Unit. The program was developed originally in BASIC on a TEKTRONIX 4051 (24K) minicomputer with CRT display. Copies of the BASIC version of the program may be obtained from the author.

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00100C       ****** SYMMETRY VERIFICATION PROGRAM ******
00110C
00120C       WRITTEN BY  JOHN B. BRADY  DURING AUGUST 1977
00130C       RUNS ON THE CONTROL DATA CORPORATION CYBER 70 SERIES COMPUTER
00140C       AT THE UNIVERSITY OF MASSACHUSETTS, AMHERST, MASSACHUSETTS
00150C
00160C
00170C THE FIRST FOUR STATEMENTS INITIALIZE THE PROGRAM AND PLOTTING
00180C PACKAGE ON THE UNIVERSITY OF MASSACHUSETTS' COMPUTER
00190        PROGRAM SYMMTRY(INPUT,OUTPUT,TAPE8=INPUT,TAPE9=OUTPUT)
00200        USE(PLOTTEK)
00210        USE(PLOTGEN)
00220        CALL PLOTS(XDUM,YDUM)
00230        DIMENSION R1(2,2),RM1(2,2)
00240        COMMON S1(2,5,12),IS,X,Y,G2,C3,R2(2,3),O5,O6
00250C ANY FIVE OR FEWER POINTS MAY BE CONNECTED TO FORM A CLOSED FIGURE
00260C FOR USE IN THIS PROGRAM.  THE DATA LIST BELOW GIVES THE COORDINATES
00270C OF 12 SUCH FIVE-POINT FIGURES.  OTHER SYMBOLS MAY BE SUBSTITUTED
00280C FOR THESE BY MAKING APPROPRIATE SUBSTITUTIONS IN THE DATA LIST.
00290        DATA S1/0.,20.,17.3205,-10.,-17.3205,-10.,0.,20.,0.,20.,20.,
00300+        4.,33.,4.,20.,14.,20.,4.,20.,4.,22.,2.,32.,2.,22.,20.,22.,2.,
00310+        22.,2.,10.,10.,10.,-10.,-10.,-10.,10.,10.,10.,20.,20.,5.,
00320+        -20.,-20.,-20.,-5.,20.,20.,20.,-2.,15.,20.,-12.,-10.,-10.,-15.,
00330+        10.,-2.,15.,18.,20.,20.,18.,-18.,-20.,-20.,-18.,18.,20.,25.,16.,
00340+        34.,16.,29.,2.,34.,16.,25.,16.,16.,0.,22.,10.,28.,0.,22.,-10.,
00350+        16.,0.,0.,20.,19.021,6.18,11.756,-16.18,-11.756,-16.18,-19.021,
00360+        6.18,25.,3.,35.771,10.826,22.457,10.826,33.228,3.,29.114,15.663,
00370+        8.,20.,-8.,20.,-8.,33.,0.,20.,8.,33./
00380        CALL PAGE
00390        PRINT 10
00400    10  FORMAT(/15X,*SYMMETRY VERIFICATION PROGRAM*
00410+        //*THIS PROGRAM WILL ENABLE YOU TO INVESTIGATE CERTAIN ASPECTS*
00420+        /*OF SYMMETRY IN TWO DIMENSIONS. THE SYMMETRY ELEMENTS*
00430+        /*AVAILABLE IN TWO DIMENSIONS ARE N-FOLD ROTATION AXES (360/N*
00440+        /*DEGREES PER ROTATION), MIRROR PLANES, AND GLIDE PLANES.  THE*
00450+        /*PROGRAM IS DESIGNED TO ALLOW YOU TO*
00460+        //3X,*1) REPEAT A SYMBOL USING SYMMETRY ELEMENTS OF YOUR*
00470+        /6X,*CHOOSING TO CREATE A TWO-DIMENSIONAL MOTIF;*
00480+        /3X,*2) TRANSLATE YOUR MOTIF TO FORM A 2-DIMENSIONAL PATTERN;*
00490+        /3X,*3) EXAMINE THE RELATIONSHIPS BETWEEN YOUR PATTERN AND*
00500+        /6X,*VARIOUS POSSIBLE LATTICES AND UNIT CELLS;*
00510+        /3X,*4) TEST THE PATTERN FOR SYMMETRY AND LOCATE THE SYMMETRY*
00520+        /6X,*ELEMENTS.*)
00530        PRINT 11
00540    11  FORMAT(/*TO PROCEDE FOLLOWING EACH PART OF THE PROGRAM, TYPE*
00550+        /*ANY INTEGER AND PRESS RETURN.*//)
00560        READ 12, IQ
00570    12  FORMAT(I3)
00580    90  CALL PAGE
00590        PRINT 13
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00600    13  FORMAT(13X,*SYMMETRY VERIFICATION PROGRAM*
00610+       /11X,*PART 1.  PLANE POINT GROUP SYMMETRY*
00620+       //*THE COMPUTER WILL CONSTRUCT A SYMMETRICAL MOTIF ACCORDING*
00630+       /*TO YOUR INSTRUCTIONS USING ONE OF SEVERAL POSSIBLE SYMBOLS.*)
00640        A=0
00650        X=0
00660        Y=0
00670        G2=0
00680C ROUTINE TO DISPLAY AVAILABLE SYMBOLS
00690C PLOT,FACTOR, AND NUMBER ARE CALCOMP SUBROUTINES.
00700        CALL FACTOR(0.01)
00710        CALL PLOT(75.,450.,-3)
00720        DO 100 IS=1,12
00730        A=IS
00740        CALL NUMBER(-50.,0.,10.,A,0.,0)
00750        CALL DRAW
00760        CALL PLOT(100.,0.,-3)
00770        IF (IS.NE.6) GO TO 100
00780        CALL PLOT(-600.,-100.,-3)
00790   100  CONTINUE
00800        CALL PLOT(-275.,-50.,-3)
00810        CALL PLOT(0.,0.,999)
00820        CALL FACTOR(0.06)
00830        PRINT 14
00840    14  FORMAT(//*WHAT SYMBOL WOULD YOU LIKE TO USE (ENTER THE APPROPRIATE*
00850+       /*NUMBER AND PRESS RETURN) ?*)
00860        READ, IS
00870        PRINT 15
00880    15  FORMAT(/*WHAT N-FOLD ROTATION AXIS WOULD YOU LIKE FOR YOUR MOTIF?*
00890+       /*TYPE N (A NON-ZERO INTEGER) AND PRESS RETURN.  FOR NO*
00900+       /*ROTATION TYPE 1 AND PRESS RETURN.*)
00910        READ, IR
00920        PRINT 16
00930    16  FORMAT(/*DO YOU WANT THE MOTIF TO INCLUDE A MIRROR PLANE?*
00940+       /*(YES=1, NO=2)*)
00950        READ, IM
00960        PRINT 17
00970    17  FORMAT(/*WOULD YOU RATHER HAVE A GLIDE PLANE ONLY (YES=1,NO=2)?*)
00980        READ, IG
00990        IF (IG.NE.1) GO TO 110
01000        G2=85/2
01010        IR=2
01020        IM=2
01030C DETERMINE (2 X 2) MATRIX FOR ROTATION OPERATOR
01040   110  R=FLOAT(IR)
01050        R=(3.1415926)*2/R
01060        R1(1,1)=COS(R)
01070        R1(1,2)=-SIN(R)
01080        R1(2,1)=-R1(1,2)
01090        R1(2,2)=R1(1,1)
01100        IF (IG.NE.1) GO TO 115
01110        R1(2,2)=1.0
01120C INITIALIZE MATRIX FOR VERTICAL MIRROR
01130   115  RM1(1,1)=-1.0
01140        RM1(1,2)=0.0
01150        RM1(2,1)=0.0
01160        RM1(2,2)=1.0
01170        CALL PAGE
01180        PRINT 18
01190    18  FORMAT(*THIS MOTIF WAS CONSTRUCTED BY REPEATING THE SYMBOL*
01200+       /*YOU SELECTED USING*)
01210        IF (IG.NE.1) GO TO 120
01220        PRINT 19
01230    1)  FORMAT(/22X,*A VERTICAL GLIDE PLANE*)
01240        GO TO 135
01250   120  IF (IM.NE.1) GO TO 125
01260        PRINT 20, IR
01270    20  FORMAT(22X,*A *,I2,*-FOLD ROTATION AXIS AND A VERTICAL MIRROR*)
01280        GO TO 135
01290   125  PRINT 21, IR
01300    21  FORMAT (22X,*A *,I2,*-FOLD ROTATION AXIS*)
01310   135  IF (IG.NE.1) GO TO 140
01320        CALL FACTOR(0.03)
01330        CALL MOTIF(R1,RM1,IR,IM)
01340        CALL FACTOR(0.06)
01350        GO TO 141
01360   140  CALL MOTIF(R1,RM1,IR,IM)
01370   141  CALL PLOT(-66.,-50.,-3)
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01380       CALL PLOT(0.,0.,999)
01390       READ, IQ
01400       CALL PAGE
01410       PRINT 22
01420    22 FORMAT(*WOULD YOU LIKE TO TRY ANOTHER MOTIF OR DO YOU WISH TO*
01430+       /*CONTINUE (1=ANOTHER MOTIF, 2=CONTINUE) ?*)
01440       READ, IQ
01450       IF (IQ.EQ.1) GO TO 90
01460    150 CALL PAGE
01470       PRINT 23
01480    23 FORMAT(13X,*SYMMETRY VERIFICATION PROGRAM*
01490+       /14X,*PART 2. PATTERN CONSTRUCTION*
01500+       //*THE COMPUTER WILL NOW TRANSLATE THE MOTIF YOU SELECTED*
01510+       /*TO CREATE A TWO-DIMENSIONAL PATTERN. PLEASE INFORM THE*
01520+       /*COMPUTER OF YOUR PREFERNECES CONCERNING THE DIMENSIONS*
01530+       /*OF THE PATTERN.*//*NUMBER OF (HORIZONTAL) TRANSLATIONS T1 =*)
01540       READ,N1
01550       PRINT 24
01560    24 FORMAT(*NUMBER OF TRANSLATIONS T2 =*)
01570       READ, N2
01580       PRINT 25
01590    25 FORMAT(/*YOU MUST ALSO PROVIDE THE RATIO BETWEEN THE LENGTHS*
01600+       /*OF TRANSLATIONS T1 AND T2.*
01610+       //*(LENGTH OF T2)/(LENGTH OF T1) =*)
01620       READ, T3
01630       PRINT 26
01640    26 FORMAT(//*FINALLY, YOU MUST SELECT THE ANGLE BETWEEN T1 AND T2.*
01650+       //*T2 /\ T1 (IN DEGREES) =*)
01660       READ, A1
01670       A1=A1*(3.1415926)*2/360
01680       CALL PAGE
01690       T2=85.
01700       T1=T2/T3
01710C COMPUTE SCALE FACTOR NEEDED TO FIT PATTERN ON SCREEN
01720       RL1=(N1+0.1)*T1+(N2+0.1)*T2*COS(A1)
01730       RL2=(N2+0.1)*T2*SIN(A1)
01740       IF ((RL1/130.).LT.(RL2/100.)) GO TO 155
01750       RL3=RL1/130.
01760       GO TO 156
01770    155 RL3=RL2/100.
01780    156 RL4=0.06/RL3
01790       CALL FACTOR(RL4)
01800C CENTER PATTERN BY ADJUSTING ORIGIN
01810       RL5=RL3*130.-(N1*T1+N2*T2*COS(A1))
01820       RL6=RL3*100-N2*T2*SIN(A1)
01830       RL51=(T1+T2*COS(A1)+RL5)/2.0
01840       RL61=(T2*SIN(A1)+RL6+G2)/2.0
01850       CALL PLOT(RL51,RL61,-3)
01860       CALL PATTRN(N1,N2,T1,T2,A1,R1,RM1,IR,IM)
01870       CALL PLOT(-RL51,-RL61,999)
01880       READ, IQ
01890       CALL PAGE
01900       PRINT 27
01910    27 FORMAT(*WOULD YOU LIKE TO CONSTRUCT ANOTHER PATTERN USING THE*
01920+       /*SAME MOTIF (1=ANOTHER PATTERN, 2=CONTINUE) ?*)
01930       READ,IQ
01940       IF (IQ.NE.1) GO TO 160
01950    159 CALL PLOT(-RL51,-RL61,-3)
01960       CALL FACTOR(0.06)
01970       GO TO 150
01980    160 PRINT 28
01990    28 FORMAT(//*WOULD YOU LIKE TO START OVER WITH A NEW MOTIF OR DO YOU*
02000+       /*WISH TO CONTINUE (1=ANOTHER MOTIF, 2=CONTINUE) ?*)
02010       READ, IQ
02020       IF (IQ.NE.1) GO TO 165
02030    164 CALL PLOT(-RL51,-RL61,-3)
02040       GO TO 90
02050    165 CALL PAGE
02060       PRINT 29
02070    29 FORMAT(13X,*SYMMETRY VERIFICATION PROGRAM*
02080+       /13X,*PART 3. UNIT CELL CONSTRUCTION*/
02090+       /*THE COMPUTER WILL NOW SUPERIMPOSE A SET OF UNIT CELLS ON*
02100+       /*YOUR PATTERN.  A TWO-DIMENSIONAL UNIT CELL IS A PARALLELO-*
02110+       /*GRAM FROM WHICH THE ENTIRE PATTERN CAN BE PRODUCED BY*
02120+       /*TRANSLATION ALONE.  THE SIDES OF THE UNIT CELL ABOUT TO*
02130+       /*BE DRAWN HAVE THE DIMENSIONS AND ORIENTATIONS OF THE*
02140+       /*TRANSLATIONS T1 AND T2 USED IN PART 2 OF THIS PROGRAM.  THE*
02150+       /*ORIGIN IS TAKEN TO BE THE ROTATION AXIS OF THE ORIGINAL*
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02160+       /*MOTIF.*)
02170        PRINT 30
02180    30  FORMAT(/*CAREFULLY OBSERVE THE POSITION OF THIS UNIT CELL, FOR IT*
02190+       /*DEFINES A 'STANDARD' COORDINATE SYSTEM THAT WILL BE USED*
02200+       /*LATER IN THIS PROGRAM.*/
02210+       /*DO YOU WANT TO SEE THE 'STANDARD' UNIT CELL OR DO YOU WISH*
02220+       /*TO CONTINUE (1=STANDARD CELL, 2=CONTINUE) ?*)
02230        READ, IQ
02240        IF (IQ.NE.1) GO TO 170
02250        W5=1
02260        X5=0
02270        Y5=0
02280        Z5=1
02290        O1=0
02300        O2=0
02310        CALL PAGE
02320        CALL PATTRN(N1,N2,T1,T2,A1,R1,RM1,IR,IM)
02330        CALL UNTCEL(W5,X5,Y5,Z5,O1,O2,T1,T2,A1,N1,N2)
02340        CALL PLOT(-RL51,-RL61,999)
02350        READ, IQ
02360    170 CALL PAGE
02370        PRINT 31
02380    31  FORMAT(*THE 'STANDARD' UNIT CELL IS ONLY ONE OF AN INFINITE NUMBER*
02390+       /*OF POSSIBLE UNIT CELLS FOR YOUR PATTERN.  YOU MAY INSTRUCT*
02400+       /*THE COMPUTER TO DRAW OTHER UNIT CELLS ON YOUR PATTERN IF*
02410+       /*YOU WISH.*//*DO YOU WANT TO DRAW ANOTHER UNIT CELL OR DO YOU *
02420+       /*WISH TO CONTINUE (1=ANOTHER UNIT CELL, 2=CONTINUE) ?*)
02430        READ, IQ
02440        IF (IQ.NE.1) GO TO 175
02450    172 CALL PAGE
02460        PRINT 32
02470    32  FORMAT(13X,*SYMMETRY VERIFICATION PROGRAM*
02480+       /13X,*PART 3. UNIT CELL CONSTRUCTION*/
02490+       /*THE SIDES OF ANY UNIT CELL MUST BE SOME LINEAR COMBINATION*
02500+       /*(VECTOR ADDITION) OF THE TRANSLATIONS T1 AND T2 USED TO *
02510+       /*CONSTRUCT THE PATTERN.*//10X,*U1 = (W)(T1) + (X)(T2)*
02520+       /10X,*U2 = (Y)(T1) + (Z)(T2)*
02530+       //*PLEASE TELL THE COMPUTER YOUR CHOICE OF THE COEFFICIENTS*
02540+       /*W,X,Y,Z (THE 'STANDARD' UNIT CELL USED  1,0,0,1).  PLEASE*
02550+       /*SEPARATE THE COEFFICIENTS BY COMMAS.*)
02560        READ, W5,X5,Y5,Z5
02570        PRINT 33
02580    33  FORMAT(//*YOU MUST ALSO SELECT AN ORIGIN FOR YOUR UNIT CELL.  THIS*
02590+       /*ORIGIN IS GIVEN IN TERMS OF THE COORDINATE SYSTEM DEFINED*
02600+       *BY THE STANDARD UNIT CELL.*//10X,*ORIGIN = (O1)(T1) + (O2)(T2)*
02610+       //*PLEASE TELL THE COMPUTER YOUR CHOICE OF THE COEFFICIENTS*
02620+       /*O1 AND O2 (THE 'STANDARD' UNIT CELL HAS ITS ORIGIN AT 0,0).*)
02630        READ, O1,O2
02640        CALL PAGE
02650        RL7=RL4/1.5
02660        CALL FACTOR(RL7)
02670        CALL PATTRN(N1,N2,T1,T2,A1,R1,RM1,IR,IM)
02680        CALL UNTCEL(W5,X5,Y5,Z5,O1,O2,T1,T2,A1,N1,N2)
02690        CALL FACTOR(RL4)
02700        CALL PLOT(-RL51,-RL61,999)
02710        READ, IQ
02720        CALL PAGE
02730        PRINT 34
02740    34  FORMAT(*WOULD YOU LIKE TO SELECT ANOTHER UNIT CELL OR WOULD*
02750+       /*YOU RATHER CONTINUE (1=ANOTHER UNIT CELL, 2=CONTINUE) ?*)
02760        READ, IQ
02770        IF (IQ.EQ.1) GO TO 172
02780        PRINT 35
02790    35  FORMAT(//*WOULD YOU LIKE TO START WITH A NEW PATTERN OR DO YOU*
02800+       /*WISH TO CONTINUE (1=ANOTHER PATTERN, 2=CONTINUE) ?*)
02810        READ, IQ
02820        IF (IQ.EQ.1) GO TO 159
02830        PRINT 36
02840    36  FORMAT(//*WOULD YOU LIKE TO START AT THE BEGINNING WITH A NEW*
02850+       /*MOTIF OR DO YOU WISH TO CONTINUE (1=NEW MOTIF, 2=CONTINUE) ?*)
02860        READ, IQ
02870        IF (IQ.EQ.1) GO TO 164
02880    175 CALL PAGE
02890        PRINT 37
02900    37  FORMAT(13X,*SYMMETRY VERIFICATION PROGRAM*
02910+       /16X,*PART 4. PATTERN SYMMETRY*/
02920+       /*YOU NOW HAVE THE OPPORTUNITY TO TEST YOUR PATTERN FOR THE*
02930+       /*PRESENCE OF SYMMETRY ELEMENTS (ROTATION AXES, MIRROR PLANES,*
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02940+       /*OR GLIDE PLANES) AT SPECIFIC LOCATIONS.  FOLLOWING YOUR*
02950+       /*INSTRUCTIONS, THE COMPUTER WILL ROTATE, REFLECT, OR GLIDE*
02960+       /*THE PATTERN AND SUPERIMPOSE THE RESULT ON THE ORIGINAL*)
02970        PRINT 38
02980    38  FORMAT(*PATTERN.  IF THE SYMMETRY ELEMENT IN QUESTION IS PRESENT,*
02990+       /*THE TWO VERSIONS OF THE PATTERN WILL SUPERIMPOSE EXACTLY;*
03000+       /*IF ABSENT, THERE WILL BE OBVIOUS MISALLIGNMENT.*/
03010+       /*WHICH KIND OF SYMMETRY ELEMENT DO YOU WISH TO TEST*
03020+       /*(1=ROTATION AXIS, 2=MIRROR PLANE, 3=GLIDE PLANE) ?*)
03030        READ, IQQ
03040        IF (IQQ.NE.1) GO TO 180
03050        PRINT 39
03060    39  FORMAT(/*WHAT N-FOLD ROTATION AXIS WOULD YOU LIKE TO TEST?*
03070+       /*N=*)
03080        READ, R7
03090        R7=(3.1415926)*2/R7
03100C DETERMINE (2 X 2) MATRIX TO ROTATE PATTERN
03110        R2(1,1)=COS(R7)
03120        R2(1,2)=-SIN(R7)
03130        R2(2,1)=-R2(1,2)
03140        R2(2,2)=R2(1,1)
03150        R2(1,3)=0
03160        R2(2,3)=0
03170        PRINT 40
03180    40  FORMAT(/*PLEASE TELL THE COMPUTER THE LOCATION (IN TERMS OF THE*
03190+       /*'STANDARD' UNIT CELL COORDINATE SYSTEM) OF THE ROTATION*
03200+       /*AXIS YOU WISH TO TEST.*//10X,*POSITION = (X)(T1) + (Y)(T2)*/
03210+       /*INPUT X AND Y SEPARATED BY A COMMA.*)
03220        READ, X7,Y7
032300       O5=X7*T1+Y7*T2*COS(A1)
03240        O6=Y7*T2*SIN(A1)
03250        GO TO 200
03260    180 IF (IQQ.NE.2) GO TO 190
03270        PRINT 41
03280    41  FORMAT(/*TO TEST FOR A MIRROR PLANE YOU MUST IDENTIFY THE INTER-*
03290+       /*CEPTS OF THAT MIRROR PLANE ON THE TRANSLATIONS T1 AND T2 (THE*
03300+       /*AXES OF THE 'STANDARD' COORDINATE SYSTEM).  IF THE MIRROR*
03310+       /*IS PARALLEL TO AN AXIS, ENTER -10 FOR THE INTERCEPT.*
03320+       //*INTERCEPT ON T1 = (X)(T1)  ; X = *)
03330        READ, X7
03340        PRINT 42
03350    42  FORMAT(//*INTERCEPT ON T2 = (Y)(T2)  ; Y = *)
03360        READ, Y7
03370        GO TO 195
03380    190 PRINT 43
03390    43  FORMAT(/*TO TEST FOR A GLIDE PLANE YOU MUST IDENTIFY THE INTERCEPTS*
03400+       /*OF THAT GLIDE PLANE ON THE TRANSLATIONS T1 AND T2.  IF THE*
03410+       /*GLIDE PLANE IS PARALLEL TO ONE OF THESE TRANSLATIONS,*
03420+       /*ENTER -10 AS THE INTERCEPT VALUE.*
03430+       //*INTERCEPT ON T1 = (X)(T1)  ; X = *)
03440        READ, X7
03450        PRINT 44
03460    44  FORMAT(//*INTERCEPT ON T2 = (Y)(T2)  ; Y = *)
03470        READ, Y7
03480        CALL PAGE
03490        PRINT 45
03500    45  FORMAT(*YOU MUST ALSO TELL THE COMPUTER THE AMOUNT OF TRANSLATION*
03510+       /*(GLIDE) PROVIDED BY YOUR GLIDE PLANE.  THIS IS MOST EASILY*
03520+       /*DONE BY GIVING THE TRANSLATION VECTOR (GLIDE) IN TERMS OF*
03530+       /*THE TRANSLATIONS T1 AND T2.*
03540+       //*GLIDE TRANSLATION = (A)(T1) + (B)(T2)*/
03550+       /*INPUT THE COEFFICIENTS A AND B SEPARATED BY A COMMA.*)
03560        READ, X8,Y8
03570C DETERMINE ORIENTATION OF MIRROR OR GLIDE PLANE
03580    195 IF (X7.EQ.-10.) GO TO 197
03590        IF (Y7.EQ.-10.) GO TO 198
03600        O5=X7*T1
03610        O6=0
03620        V1=X7*T1-Y7*T2*COS(A1)
03630        V3=Y7*T2*SIN(A1)
03640        V2=SQRT(V1**2+V3**2)
03650        A3=ACOS(V1/V2)
03660        A4=2.0*(3.1415926/2.0-A3)
03670        GO TO 199
03680    197 O5=0
03690        O6=Y7*T2*SIN(A1)
03700        V1=T1
03710        V3=0
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03720        A4=3.1415916
03730        GO TO 199
03740    198 O5=X7*T1
03750        O6=0
03760        V1=-T2*COS(A1)
03770        V3=T2*SIN(A1)
03780        A4=2.0*(A1-3.1415926/2.0)
03790C DETERMINE (2 X 2) MATRIX FOR MIRROR OF THIS ORIENTATION
03800    199 R2(1,1)=-COS(A4)
03810        R2(1,2)=-SIN(A4)
03820        R2(2,1)=R2(1,2)
03830        R2(2,2)=-R2(1,1)
03840        R2(1,3)=0
03850        R2(2,3)=0
03860    200 CALL PAGE
03870        IF (IQQ.NE.3) GO TO 205
03880        R2(1,3)=X8*T1+Y8*T2*COS(A1)
03890        R2(2,3)=Y8*T2*SIN(A1)
03900    205 CALL PATTRN(N1,N2,T1,T2,A1,R1,RM1,IR,IM)
03910        C3=1
03920        IF (IQQ.NE.1) GO TO 210
03930C PLOT LOCATION OF ROTATION AXIS TO BE TESTED
03940        CALL PLOT(O5+5,O6,3)
03950        CALL PLOT(O5-5,O6,2)
03960        CALL PLOT(O5,O6+5,3)
03970        CALL PLOT(O5,O6-5,2)
03980        GO TO 220
03990C PLOT LOCATION OF MIRROR OR GLIDE TO BE TESTED
04000    210 CALL PLOT(O5+5*V1,O6-5*V3,3)
04010        DO 215 I=1,80
04020        RI3=FLOAT(I)/5.0
04030        CALL PLOT(O5-(RI3-6.0)*V1,O6+(RI3-6.0)*V3,2)
04040        IF (IQQ.NE.3) GO TO 215
04050        CALL PLOT(O5-(RI3-5.9)*V1,O6+(RI3-5.9)*V3,3)
04060    215 CONTINUE
04070    220 CALL PATTRN(N1,N2,T1,T2,A1,R1,RM1,IR,IM)
04080        CALL PLOT(-RL5,-RL6,999)
04090        C3=0
04100        READ, IQ
04110        CALL PAGE
04120        PRINT 46
04130     46 FORMAT(*WOULD YOU LIKE TO TEST YOUR PATTERN FOR ANOTHER SYMMETRY*
04140+        /*ELEMENT (1=ANOTHER TEST, 2=CONTINUE) ?*)
04150        READ, IQ
04160        IF (IQ.EQ.1) GO TO 175
04170        PRINT 47
04180     47 FORMAT(/*WOULD YOU LIKE TO EXAMINE ANOTHER UNIT CELL (1=ANOTHER*
04190+        /*CELL, 2=CONTINUE) ?*)
04200        READ, IQ
04210        IF (IQ.EQ.1) GO TO 165
04220        PRINT 48
04230     48 FORMAT(/*WOULD YOU LIKE TO TRY ANOTHER PATTERN WITH THE SAME*
04240+        /*MOTIF (1=ANOTHER PATTERN, 2=CONTINUE) ?*)
04250        READ, IQ
04260        IF (IQ.EQ.1) GO TO 159
04270        PRINT 49
04280     49 FORMAT(/*WOULD YOU LIKE TO START ALL OVER AGAIN OR DO YOU WANT*
04290+        /*TO GO HOME (1=AGAIN, 2=GO HOME) ?*)
04300        READ, IQ
04310        IF (IQ.EQ.1) GO TO 164
04320        PRINT 50
04330     50 FORMAT(///*GOODBYE!*)
04340        END
04350        SUBROUTINE DRAW
04360        DIMENSION S3(2,5)
04370        COMMON S1(2,5,12),IS,X,Y,G2,C3,R2(2,3),O5,O6
04380        DO 579 I=1,5
04390        S3(1,I)=S1(1,I,IS)+X
04400    579 S3(2,I)=S1(2,I,IS)+Y
04410        IF (C3.NE.1.0) GO TO 584
04420C REORIENTATION ROUTINE FOR PART 4
04430        DO 583 I=1,5
04440        S3(1,I)=S3(1,I)-O5
04450        S3(2,I)=S3(2,I)-O6
04460        X5=R2(1,1)*S3(1,I)+R2(1,2)*S3(2,I)+R2(1,3)+O5
04470        Y5=R2(2,1)*S3(1,I)+R2(2,2)*S3(2,I)+R2(2,3)+O6
04480        S3(1,I)=X5
04490        S3(2,I)=Y5
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04500    583 CONTINUE
04510    584 CALL PLOT(S3(1,5),S3(2,5),3)
04520        DO 587 I=1,5
04530        CALL PLOT(S3(1,I),S3(2,I),2)
04540    587 CONTINUE
04550        CALL PLOT(0.,0.,3)
04560        RETURN
04570        END
04580        SUBROUTINE MOTIF(R1,RM1,IR,IM)
04590        DIMENSION S2(2,5),R1(2,2),RM1(2,2)
04600        COMMON S1(2,5,12),IS,X,Y,G2,C3,R2(2,3),05,06
04610        IM1=0
04620    591 DO 600 I=1,IR
04630        IF (IM1.EQ.1) GO TO 592
04640        CALL DRAW
04650    592 DO 595 J=1,2
04660        DO 595 K=1,5
04670    595 S2(J,K)=S1(J,K,IS)
04680        IF (IM1.NE.1) GO TO 597
04690        CALL PRODUC(RM1,S2)
04700        CALL DRAW
04710    597 G2=-G2
04720        CALL PRODUC(R1,S2)
04730    600 CONTINUE
04740        IF (IM.NE.1) GO TO 605
04750        IF (IM1.EQ.1) GO TO 605
04760        IM1=1
04770        GO TO 591
04780    605 RETURN
04790        END
04800        SUBROUTINE PRODUC(D1,S2)
04810        DIMENSION D1(2,2),S2(2,5)
04820        COMMON S1(2,5,12),IS,X,Y,G2,C3,R2(2,3),05,06
04830        DO 610 J=1,5
04840        S1(1,J,IS)=D1(1,1)*S2(1,J)+D1(1,2)*S2(2,J)
04850    610 S1(2,J,IS)=D1(2,1)*S2(1,J)+D1(2,2)*S2(2,J)+G2
04860        RETURN
04870        END
04880        SUBROUTINE PATTRN(N1,N2,T1,T2,A1,R1,RM1,IR,IM)
04890        DIMENSION R1(2,2),RM1(2,2)
04900        COMMON S1(2,5,12),IS,X,Y,G2,C3,R2(2,3),05,06
04910        DO 640 I=1,N2
04920        DO 640 J=1,N1
04930        X=(J-1)*T1+(I-1)*T2*COS(A1)
04940        Y=(I-1)*T2*SIN(A1)
04950        CALL MOTIF(R1,RM1,IR,IM)
04960    640 CONTINUE
04970        RETURN
04980        END
04990        SUBROUTINE UNTCEL(W5,X5,Y5,Z5,01,02,T1,T2,A1,N1,N2)
05000        COMMON S1(2,5,12),IS,X,Y,G2,C3,R2(2,3),05,06
05010        U1=W5*T1+X5*T2*COS(A1)
05020        U2=X5*T2*SIN(A1)
05030        U3=Y5*T1+Z5*T2*COS(A1)
05040        U4=Z5*T2*SIN(A1)
05050        03=01*T1+02*T2*COS(A1)
05060        04=02*T2*SIN(A1)
05070        N3=N1
05080        N4=N2-1
05090        DO 650 I=1,N4
05100        CALL PLOT(((I-1)*U3+03),((I-1)*U4+04),3)
05110        DO 650 J=1,N3
05120        CALL PLOT(((J-1)*U1+(I-1)*U3+03),((J-1)*U2+(I-1)*U4+04),2)
05130        CALL PLOT(((J-1)*U1+I*U3+03),((J-1)*U2+I*U4+04),2)
05140        CALL PLOT(((J-1)*U1+(I-1)*U3+03),((J-1)*U2+(I-1)*U4+04),3)
05150    650 CONTINUE
05160        CALL PLOT((N4*U3+03),(N4*U4+04),3)
05170        CALL PLOT(((N3-1)*U1+N4*U3+03),((N3-1)*U2+N4*U4+04),2)
05180        RETURN
05190        END
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