

Index

A

\mathcal{A} (annulus), 11
 \mathcal{A} (strip, covering of the annulus), 11
action, 19, 146, 247
action-angle variables, 79
action minimizer, 36
 α -asymptotic, 276
 α limit set, $\alpha(x)$, 49
annulus, 11
anti-integrable, 197
Aubry diagram, 35
Aubry-Mather sets, 32, 44, 44
attaching a cell, 260
attaching map, 260
average action, 205

B

base, 12
 β (Mather's minimal action function), 206
Betti number, 266
billiard map, 25
Birkhoff orbit, 34
boundary twist condition, 13, 132
bouquet of spheres, 262
broken geodesic, 152

C

\mathcal{A} (cylinder) 11
canonical,

- transformations, 256
- symplectic form, 234

Cantor set, 50
cantorus, cantori, 45
cell, 260
cellular space, 260
chain complex, 264
CO, cyclically ordered, 34
cyclically ordered orbits, 34
coercive function, coercion condition, 32
configuration space, 233
cohomology, 267
cohomology Conley index, 274, 278
coisotropic,

- submanifold, 242
- subspace, 236

conjugate,

- coordinates, 243
- topologically –, 33

Conley index, 278
contractible, 289
convexity

- of Lagrangians, 200

completely integrable, 1, 22, 239
completeness (of E-L flow), 200
cotangent,

- bundle, 243
- vector, 243

covering,

- map, 11, 288

– space, 288

– transformation, 289

critical point, 267

crossing of sequences, 35, 56

cuplength, 274

cup product, 267

CW complex, 260

cyclically ordered, CO, 34

cylinder, 11

D

deck transformation, 289
degree, 11, 262
 δ_{jk} , Kroenecker symbol, 243
DeRham cohomology group, 267
dimension,

- of a cell complex, 260

distance, 148

E

elliptic
exact Lagrangian, 135
exact symplectic,

- form, 245
- manifold, 245
- map 15, 245

exit set, 274
exponential map, 147
extended phase space, 251
extreme points, 204

F

factor, 33
fiber, 12

- critical points (manifold of), 225
- Intersection Property, 187

filtration, 278
finite cell complex, 260
flux, 14
first integral, 252
F-ordered, 41
free homotopy, 184
fundamental group, 289

G

generalized Mather sets, 207
generating function

- of a twist map, 17
- of a symplectic twist map, 99
- of a Lagrangian manifold, 247, 285

generating phase, 284, 285
 – for Lagrangian manifolds, 225, 247
 – quadratic at infinity, 285
 generic, 112
 geodesic, 152
 – flow, 147
 ghost torus, 117
 gpqi, 285
 gradient, 268
 gradient flow, 268

H

Hamiltonian,
 – equations, 249, 251
 – function, 248
 – map, 251
 – vector field, 250, 251
 height, 71
 heteroclinic orbit, 49
 homoclinic orbit, 49
 homology group, 264
 homotopic, 263
 homotopy, 263
 – formula, 253
 – type, 266
 hyperbolic fixed point and
 periodic orbit, 23, 239
 hyperbolic manifold, 152

I

index pair, 278
 induced, 12
 injectivity radius, 177
 integral invariant of Poincaré, 255
 intersection index, 56
 involutive, 249
 isolating block, 274
 isolated invariant set, 278
 isotopy, 251
 isotopic to Id, 16
 isotropic,
 – subspace, 236
 – submanifold, 242

J

Jacobi matrix, 64

K

Kronecker Symbol, δ_{jk} , 243

L

Lagrangian,
 – function, 247
 – subspace, 236
 – submanifold, 242
 Legendre
 – condition, 154, 154, 248

– transformation, 248
 Lie derivative,
 – of a function 252
 – of a form, 253
 lift, 288
 linear domain, 207
 – embedding condition, 154
 global – condition, 154 – trans-
 formation, 248
 Liouville form, 244
 linking number, 189
 linking of spheres, 189

M

Maslov
 – class, 133
 – index, 232
 maximal
 – invariant set, 274
 – skeleton, 65
 minimally linked (skeleton), 68
 minimal measure, 206
 minimal set, 49
 minimizer,
 – for twist maps, 36
 – for a Lagrangian system, 201
 minimizing orbits, segments, 36
 (m, n),
 – minimizer, 36
 –(periodic) orbits, 20
 – sequences, 20
 m_d , d-periodic point, 103
 \mathcal{M}_L (E-L invariant measures), 205
 monotone orbits, 34
 – of the gradient flow, 63
 monotone flow, 53
 monotone recurrence relation, 196
 Morse decomposition, 276
 Morse function, 268
 Morse index, 268
 Morse-Smale function, 272

N

nondegenerate
 –periodic orbit, 111
 –critical point, 267
 normally hyperbolic (invariant set), 283
 null (subspace), 236

O

ω -asymptotic, 276
 Omega limit set $\omega(x)$, 49
 optical Hamiltonian system, 159
 orbit, 18
 order interval (in sequence space), 55
 orthogonal subspace, 234

P

parabolic, 239
 partial order (on sequences), 34
 periodic action, W_{mn} , 20

PCO_ω , periodic CO sequences 73
phonon gap, 120
 $\pi_1(M, z_0)$, $\pi_1(M)$, 289
Poincaré–Melnikov function, 139
Poincaré–Melnikov , 139
Poincaré polynomial, 278
pointwise convergence, 34
Poisson bracket, 252
polar decomposition, 241
Positive cone, 55
principle of least action, 247
proj (projection from covering space), 11
pseudograph, 130
 ψ (twist change of coordinates), 13
pull-back (of a map), 243

R

ratchet phenomenon, 41
recurrent, 39
reduced symplectic space, 229
region of instability, 141
related by continuation, 283
residual, 112
residue, 119
retraction, 280
Riemannian manifold, 146
rotation number, 21

- backward and forward –, 21
- of a point, 21
- of a ghost circle, 60

rotation vector

- of a curve on a manifold, 205
- of a curve on a hyperbolic manifold, 214
- of an invariant measure, 203, 204
- of a periodic orbit of a map, 103
- of a sequence, 103

S

\mathbb{S}^1 , circle, 11
semiconjugacy, 33
shift map, 54
shear map, 22
simply connected, 289
skeleton

- of critical sequences, 65
- of a CW-complex, 260

space average, 203
stable,

- fixed points, 239
- manifold, 239
- norm, 210

stabilization (of gpqi's), 286
standard map, 21

- generalized –(on hyperbolic manifolds), 153

strip, 11
sublevel sets, 269
subsolution (for a monotone recurrence relation), 196
sum of the Betti numbers, 274
superlinearity, 200

superquadraticity, 213
support (of a measure), 202
symplectic

- capacity, 231
- form, 15, 233, 241
- group, 238
- isotopy, 253
- map, 15, 236, 238
- structure, 241
- vector space, 234, 236
- twist map, 3, 88, 99
- reduction, 230

T

T, integer translation, 11
 \mathbb{T}^n , torus, 88
tangency (of sequences), 56
tilt map, 20
time average, 201
Tonelli's theorem, 201
topological stability, 33
trace, 119
transverse intersection

- of sequences, 56
- of manifolds, 225

trimming, 130
twist condition,

- for annulus maps, 1, 12
- vs. gen. functions, 18
- for maps of \mathbb{T}^n , 88
- for maps of T^*M , 99

twist map

- of the annulus, 12
- positive or negative, 13

symplectic, 3, 88, 99

U

U_L , 206
unitary group, 240
unit cotangent bundle, 147
universal cover, 289
universal abelian cover, 290
unstable manifold, 269, 239

V

Variation equation, 160

W

weak* limit, 203
well ordered (orbit), 34
 W_{mn} , 20

- minimizer, 36

X

$X_{m,n}$, (m,n sequences), 20
 $\overline{\overline{X}}$, (m, dN sequences), 107
 \overline{X} , quotient of $\overline{\overline{X}}$, 108