Math 343, Topics in Analysis **Final** **Prof. Golé, May 2000**

The total number of points for this test is 39, with 9 extra credit. The exam is due by Friday, May 5 at 4 pm. Either hand it to me if I'm around, or to the secretaries in the science office. Have a good summer if I don't see you!

Problem 1 (8 pts.)
Let $f : [0,1] \to [0,1]$ be continuous. Prove that f has a fixed point.

Problem 2 (8 pts.)
Let $F : \mathbb{R}^n \to \mathbb{R}^n$ be of class C^1 and have nonzero Jacobian at every point. Show that $F(\mathbb{R}^n)$ is open.

Problem 3 (8 pts.)
Show that the equations:

$$\begin{cases} x^3 + y^{11} & = 2 \\ xz + y^2 + y & = 3 \end{cases} .$$

are solvable for y, z in terms of x near the point $(1,1,1)$. Compute the derivatives $dy/dx, dz/dx$ at that point.

Problem 4 (15 +3 xtra credit pts.)
Which of the following sets are open, close, compact, connected?
a) $f : \mathbb{R}^2 \to \mathbb{R}$ continuous, $A = \{(x,y) \in \mathbb{R}^2 \mid f(x,y) > 3\}$
b) $B = \{(x^2 - 3x + yx,\ yx^2) \in \mathbb{R}^2 \mid \sup\{|x|, |y|\} \leq 1\}$
c) $f : \mathbb{R}^2 \to \mathbb{R}$, continuous, $C = \{f(x,y) \in \mathbb{R} \mid x^2 + y^2 = 1\}$
d) (Extra credit) $f : \mathbb{R}^2 \to \mathbb{R}$, continuous $C = \{(x, y, f(x,y)) \in \mathbb{R} \mid x^2 + y^2 = 1\}$.
In (a), (c) and (d), if a property is true for all continuous f, prove it. If not, give counterexamples that show a case where it is true, another where it is false.

Problem 5 (6 extra credit pts.)
For which value of a is the following equation **not** representing a manifold:

$$x^2 + y^2 + 2y - z^2 = a - 1$$

Any idea of what it looks like for that value of a?