This final is due back to me on Friday, December 22. Although some questions are interrelated, you can do most of them independently. Very few questions warrant large computations, so look for simple answers first. Do your best!

Problem 1 (3, 5, 5)
Let S be a surface parametrized by $(s,t) \to (u(s), v(s), t),\ \ s \in [0,1), t \in \mathbb{R}$ where $(u(s), v(s))$ is a planar curve parametrized by arc length.
a) Describe S in words and pictures. Give a specific example of such a surface.
b) Show that, if you remove the line $\{(u(0), v(0), t) \mid t \in \mathbb{R}\}$ from it, the surface S is isometric to a region of the plane. Do this by exhibiting an isometry.
c) Describe all the geodesics on S.

Problem 2 (4, 2)
a) Following the clue in the last two lines of page 160 of the text, show that if S is a surface of revolution, *i.e.* the graph of some function $y = f(x)$ rotated around the x axis, a meridian through a point $(x_0, f(x_0))$ of the graph of f is a geodesic if and only if x_0 is a critical point of f (*i.e.* $f'(x_0) = 0$).
b) How does a) apply to an ellipsoid of revolution?

Problem 3 (4, 4)
a) Show that if $S \subset \mathbb{R}^3$ is a surface isometric to the plane then, through each point of S, there is at least one direction in which the normal curvature of S is zero.
b) For an S such as in a), show that through each point of S there passes a straight (euclidean) line segment. (*Hint.* Look at the end of Chapter 10.)

Problem 4 (5)
Do Exercise 10.3 and apply this formula of curvature to (re)compute the curvature of the sphere of radius R. (*Hint.* Use spherical coordinates. Do at least the case $R = 1$...)

Problem 5 (5)
What interior angle must an equilateral geodesic triangle of area 1 have on the sphere of radius R?

[Please turn over]

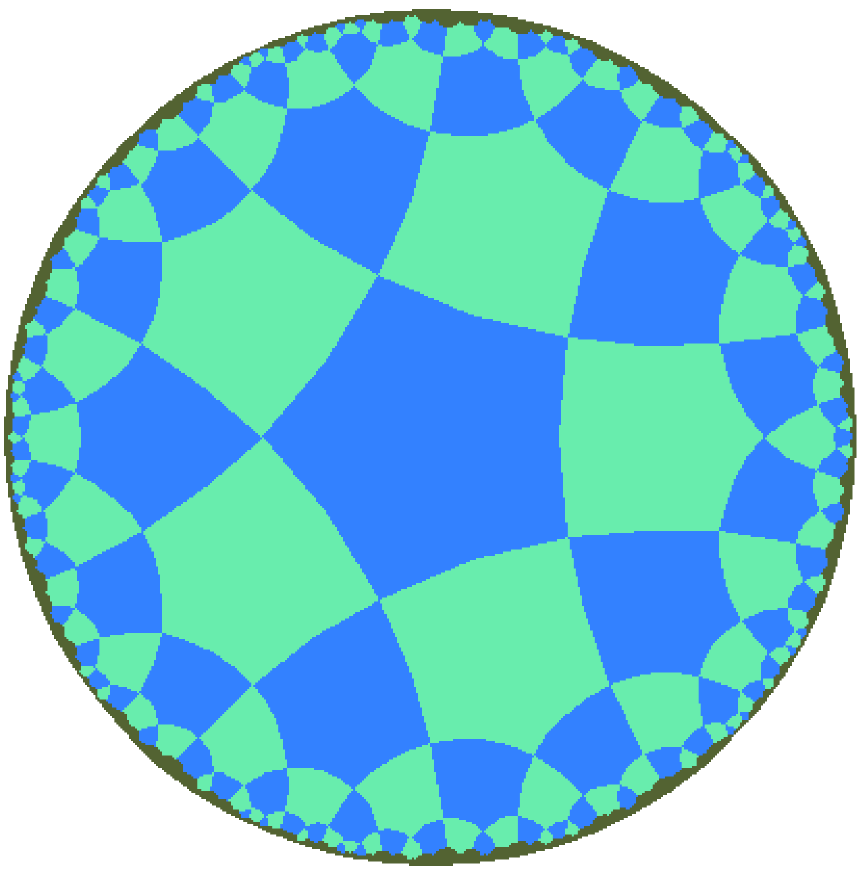
Problem 6 (5, 4, 4, 2)
A model of hyperbolic geometry is the Poincaré disk. Here we will choose the disk

$$D_2 = \{(x, y) \mid x^2 + y^2 < 4\},$$

and give it the metric

$$ds^2 = \frac{dx^2 + dy^2}{\left(1 - \frac{x^2+y^2}{4}\right)}.$$

a) Compute the Christoffel symbols for this metric.
b) Show that the Gaussian curvature κ is -1.
c) The picture below is a tiling by geodesic pentagons of the disk described above. Compute the area of the pentagon at the center of the picture.
d) What can you say about the (hyperbolic) areas of the other pentagons in the picture?



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It was fun working with you all. Have a good break!