MATH 325

In this note, I would like to describe what I intend to do in this class, and hopefully convey some of my enthusiasm about the material.

ABOUT THE MATERIAL:

Complex analysis came about last century when mathematicians tried to solve analytically the problem of heat conduction. The theory grew very fast and soon had repercussions in many Øelds of mathematics as well as of physics. There would not be Quantum mechanics without it and you would not be sitting in front of this computer either. In mathematics, Complex Analysis has very varied uses, from number theory to dynamical systems. I use it right now in my research on spiral patterns in pine cones!

In Complex Analysis, we study complex valued functions f(z) of the complex variable z. The same way we do in Calculus, we can de@ne the derivative \mathcal{P} of \mathcal{F} as a limit. When this derivative exists, \mathcal{F} is called *analytic*. This is (almost) where the analogy with Calculus stops. It turns out that analytic functions have incredibly rich properties. For instance, if a function \mathcal{F} is analytic, that is if \mathcal{P} exists, then all its successive derivatives \mathcal{P} . \mathcal{P} can be represented by power series. Thus, why and when real functions can be represented by power series is better understood in the setting of complex functions, and this understanding in turn is very helpful for the computation of improper integrals.

APPROACH TO THE MATERIAL:

I will be using \Visual Complex Analysis", a new book by T. Needham (available at the bookstore). I Ønd it a remarkable book in that it exploits to its fullest the very geometric character of analytic functions. If you look at analytic functions as transformations of the plane into itself (the complex number z = x + iycan be identi@ed with the point $(x \cdot y)$), they preserve the angles between intersecting curves. This simple property, called conformality, is what this book exploits, centering its arguments around very suggestive pictures. It turns an otherwise analytic theory into a geometric one and thus may give you a more intuitive understanding of the subject and its ties to geometry. We will use the computer program F(z) to help us in this visualization.

YOUR ROLE IN THE CLASS:

Although the word \proof" is replaced by \explanation" in the textbook, this is a pretty conceptual material, and your job will be, in the homework as well as in the take home midterm, more often to explain than to compute. Because of that, group work is highly encouraged. For the Ønal, you can either take an exam or complete a project, which can take several forms:

1) A survey paper on a theme that we could not cover in class. The material could be taken from the many facinating sections of the book that are marked with a star and intended for further reading, or from other sources.

2) A guided sequence of problems leading to an interesting result.

- 3) A computer aided exploration of some aspect of the class.
- 4) A combination of the above.

MORE INFORMATION:

Do not hesitate to come and knock at my door, Burton 316 (x 3875), or email me at

cgole@math.smith.edu. I will soon put on the web a (tentative) syllabus spelling out the material covered and the grading policy (www.math.smith.edu/ cgole)