Math 212 (01), Calculus III **Practice Final Examination** **Prof. Golé, May 2000**

For the final, you may use a (two-sided) sheet of notes and a (graphing) calculator.

Problem 1

Calculate the following integrals:

a) $\iint_R y^2 e^{xy^3}\,dA$ where R is the rectangular domain within the points $(1,3), (-1,3)$, $(1,-1)$ and $(-1,-1)$.

b) $\iint_D \sin(1-x^2-y^2)dA$ where D is the region in the second quadrant bounded by the x-axis, the line $y=\sqrt{3}x$, the circles $x^2+y^2=4$ and $x^2+y^2=9$.

Problem 2

a) Give the equation of the tangent plane to the surface of equation $z=xy$ at the point $(1,2,2)$.

b) Show that $(1,0,0)$ is one of the points of intersection of the unit circle parameterized by $\langle \cos t, \sin t, 0\rangle$ and the graph of f above.

c) Find the angle the circle makes with $z=xy$ at $(1,0,0)$.

Problem 3

Establish whether the following are true or false. Justify your answers!

a) $f_y(a,b) = \lim_{y\to b} \frac{f(y)-f(b)}{b-y}$

b) $\int_{-1}^{1}\int_{0}^{1} e^{x^2+y^2} y^3 dxdy = 0$ (No computation necessary!).

c) The line integral $\int_C 2x\sin y dx + x^2\cos y dy = 9$ where C is any curve joining the points $(33333, 0)$ and $(3, \pi/2)$

Problem 4

a) Calculate the iterated integral : $\displaystyle\int_0^2 \int_1^{y^2+1} \int_0^{x+y} (x-2y)\ dz\ dx\ dy$

b) Describe the region of integration in the previous integral.

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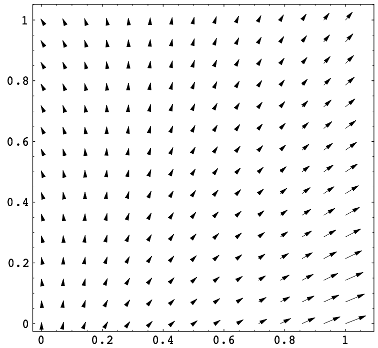
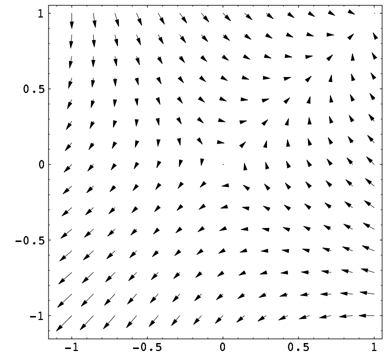
Problem 5



(1)



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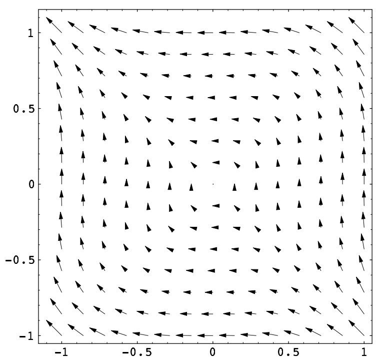

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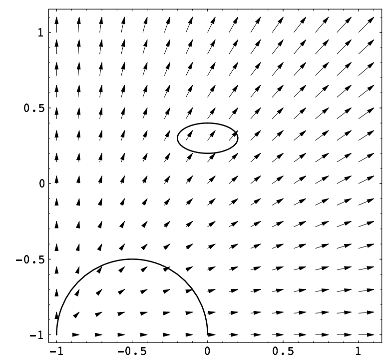

(4)

a) Draw a few flowlines for the vector fields (1) and (2).

b) Match the vector fields (1)-(4) with the following (justify your answers!):

$$\mathbf{F}(x,y) = \langle x+1, y+1\rangle \qquad \mathbf{G}(x,y) = \left\langle -x^2+y, -y^2+x\right\rangle$$
$$\mathbf{H}(x,y) = \left\langle -y^2, x^2\right\rangle \qquad \mathbf{J}(x,y) = \left\langle e^{3x-y}-1, e^{2x}\right\rangle$$

c) *(You will need to do (b) to solve this question)* Show that the line integral of the vector field (4) along the ellipse shown is 0. Compute the line integral of (4) along the upper half of circle shown, traversed clockwise.

d) Compute the line integral of **H** along the line segment between $(-1,0)$ and $(0,-1)$.

e) Compute the line integral of **J** along the rectangle with vertices $(3,1), (-1,1)$, $(3,-1)$ and $(-1,-1)$ traversed once counterclockwise.