
Problem 1.
a) Using Taylor's formula to order 3 for $\cos \pi x$, show that

$$(\cos \pi x)^2 = 1 - \pi^2 x^2 + O(x^4).$$

b) Using a) and Taylor's formula for $\sin x$ to order 3, compute $\lim_{x \to 0} \dfrac{x(\cos \pi x)^2 - \sin x}{x^3}$

Problem 2. This problem is about geometric series.
a) Find the sum of $x^2 - 2x^3 + 3x^4 - 4x^5 + \ldots$
b) Write $.11331133\overline{1133}\ldots$ as a fraction.

Problem 3. A ball is dropped from 3 feet high. Each time it rebounds, it goes 2/3 of the way up from the highest point of the last rebound. This ideal ball never stops, and we want to know how much distance it will cover until $t = \infty$
a) Start writing the distance the ball travels down as a geometric series. Find the sum of that series.
b) Now write the distance the ball travels up, modifying your series in a). Sum this one and add it to the previous one to get the total distance travelled.

Problem 4. Determine whether each of the following series converges or diverges.
a) $\displaystyle\sum_{n=0}^{\infty} \frac{4n+1}{n!}$
b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{\pi}\right)^n$
c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{3n}{n+3}$

Problem 5. Determine the set of all x such that the series converges:
a) $x + 2x^2 + 3x^3 + 4x^4 + \ldots$
b) $\displaystyle\sum_{n=0}^{\infty} \frac{3n(x+\pi)^n}{n!}$
c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} x^n$
d) $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{n} x^n$