Problem 3 (25 pts) Modeling the Growth of Population. For this problem you will find useful to (re)read Section 2.4 of the book. Similarly to Assignment # 1 (page 71), we want to find a model for the world population that fits a certain data.

Warm Up. Consider the logistic equation:

$$Y' = kY\left(1 - \frac{Y}{C}\right)$$
$$Y(0) = .1$$

a) Set $k = 3$ and $C = 2$ in this question. Using graph paper preferably, draw the slope field for this differential equation, in a rectangle where t varies between 0 and 4 and Y between 0 and 3. How can you "read off" the carrying capacity from your slope field?

b) Using Slinky (http://math.smith.edu/ cohenle/slinky/slinky.html), find the value for $Y(1.4)$.

c) Keeping $C = 2$, use Slinky to find the value of k so that $Y(2) = 1.89385$. Try to give three digits of accuracy for k.

The Real Thing.

d) Now for some real world data! A 1996 UN study has found that the world population may not be growing as fast as was expected. It suggests that the population may be stabilizing after the year 2200 at 10.73 billions. The following table shows the following past and (expected) future data. Using Slinky, find constants k and C such that the logistic model as above fits the UN data best. As in the Yeast growth data, this is real data and hence you won't find a model that fits perfectly... Nonetheless, try to get as many significant digits as possible for your k and C. Show the populations your model predict for the same years that appear on the table.

Year	Population (Billions)
1804	1
1927	2
1960	3
1974	4
1987	5
1999	6
2011	7
2025	8
2041	9
2071	10