Errata: Advanced Calculus: A Geometric View

Corrections to 21 March 2017

I wish to express my thanks to David Berkowitz for corrections and helpful comments about the text.

Corrections are marked in red, where possible.

Page 11, line 4. Change -2π to $-\frac{4}{3}\pi - \sqrt{3}$.

Page 11, line 7. Change $d\mathbf{x} = (-2\cos t, -2\sin t)\,dt$ to $d\mathbf{x} = (2\cos t, -2\sin t)\,dt$.

Page 24, Exercise 1.26. Replace "semicircle" by "circular arc" and replace the two occurrences of "0" by "1":

Let $\vec{C}$ be the circular arc of radius 2 centered at the origin, oriented counterclockwise from $(1, -\sqrt{3})$ to $(1, \sqrt{3})$.

Page 27, line -10 to end of exercises. At several places, "0" needs to be replaced by "μ". Moreover, the word "variables" needs to be made singular in two places and "deviations" in one. The text should read:

...(only for the sake of simplicity) that $\mu < a$; then

$$\text{Prob}(a \le X_{\mu,\sigma} \le b) = \text{Prob}(\mu \le X_{\mu,\sigma} \le b) - \text{Prob}(\mu \le X_{\mu,\sigma} \le a).$$

In other words, it is sufficient to calculate only $\text{Prob}(\mu \le X_{\mu,\sigma} \le b)$ for various values of b. The following is the second strategy.

1.39. Suppose $Z_{0,1}$ is a normal random variable with mean 0 and standard deviation 1. Continue to assume $X_{\mu,\sigma}$ is a normal random variable with mean μ and standard deviation σ. Show that

$$\text{Prob}(\mu \le X_{\mu,\sigma} \le b) = \text{Prob}(0 \le Z_{0,1} \le (b-\mu)/\sigma).$$

Suggestion: Consider the push-forward substitution $z = (x-\mu)/\sigma$ and use it to show that

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{\mu}^{b} \mathrm{e}^{-(x-\mu)^2/2\sigma^2}\,dx = \frac{1}{\sqrt{2\pi}} \int_{0}^{(b-\mu)/\sigma} \mathrm{e}^{-z^2/2}\,dz.$$

The last result implies that it is sufficient to calculate (e.g., by numerical integration) the values

$$P(z_0) = \text{Prob}(0 \le Z_{0,1} \le z_0)$$

for various numbers $z_0 > 0$. In other words, we need only know the distribution of one very special normal random variable, $Z_{0,1}$; all others can be calculated from it. The values $P(z_0)$ are some times called "z-scores"; the probability that a given normal random variable lies in a given range reduces to knowing certain z-scores.

1.40. For simplicity, we assumed that $a > \mu$ when we reduced probabilities for $X_{\mu,\sigma}$ to certain z-scores. This assumption is not necessary; describe how to remove it.

page 35, line -15. Delete the repeated "is".

Page 63, line -10. Delete the closing parenthesis.

Page 65, line -6. Replace "$\mathbf{e}_n$" by "$\mathbf{v}_n$".

Page 69, Exercise 2.39. The beginning of the sentence should read "Find a vector $\mathbf{h}$ that is orthogonal to $\mathbf{v}_1 \wedge \mathbf{v}_2$ and is in the plane…".

Page 72, Marginal figure. The slope of the straight line should be

$$\frac{f(b) - f(a)}{b - a}.$$

Page 72, line 4. Replace "Also, f' has to be continuous from a to b" by "For future convenience, we take f' to be continuous from a to b."

Page 104, Exercise 3.25. Springer style requires that the base of natural logarithms be written as "e", rather than "e", in "…of degree 4 for $\mathrm{e}^x \cos y$ at …". Also, change "…on page 96." to "…on pages 96–97."

Page 143, Exercise 4.11.c. The lower right term in the matrix needs an additional factor of "2"; the matrix should read

$$\mathrm{d}\mathbf{h}_{\mathbf{r}} = \begin{pmatrix} 2r & 0 \\ 2r\cos 2\theta & -2r^2 \sin 2\theta \end{pmatrix}.$$

Page 146, Exercise 4.21.c. "…the local area multiplier of $\mathbf{f}$ at (a,b) is $2b$."

Page 146, Exercise 4.23. The formula for θ is correct but expressed in a nonconventional way. By convention, the "3" should precede the"a":

$$\theta = \arctan\left(\frac{3a^2b - b^3}{a^3 - 3ab^2}\right).$$

Page 148, Exercise 4.34. The argument of the arctangent function should be "v/u", not "y/x": $\varphi = \arctan(v/u)$.

Page 148, Exercise 4.36. Replace "be" by "are" to give "…and $y = y(t)$ are differentiable…".

Page 149, Exercise 4.36.b. Replace "d" by "d" and "$d\varphi$" by $\varphi'(t)\,dt$" in the displayed formula:

$$\int_{\vec{C}} \mathbf{F} \cdot d\mathbf{x} = \int_a^b \varphi'(t)\,dt = \Phi(\mathbf{x}) \Big|_{\text{start of } \vec{C}}^{\text{end of } \vec{C}}.$$

Page 149, Exercise 4.38. In the last displayed equation, θ is to be evaluated at the start and end of $\vec{C}$, not $\mathbf{f}(\vec{C})$. Thus,

$$I = \Delta\theta = \theta \Big|_{\text{start of } \vec{C}}^{\text{end of } \vec{C}}.$$

Page 179, Exercise 5.12.b. The exercise is correct as written, but clarity and coherence require that the triple of variables r, z, θ always appear in that order. Thus

5.12b. Determine

$$\frac{\partial(\rho,\theta,\varphi)}{\partial(r,z,\theta)} \text{ and } \frac{\partial(r,z,\theta)}{\partial(x,y,z)}$$

and verify directly that

$$\frac{\partial(\rho,\theta,\varphi)}{\partial(x,y,z)} = \frac{\partial(\rho,\theta,\varphi)}{\partial(r,z,\theta)}\frac{\partial(r,z,\theta)}{\partial(x,y,z)}.$$

Page 180. Exercise 5.17.c. Modify the formula for y to indicate a multiplication, and correct the spelling of "census", thus: "…function $y = B\times 10^{kx}$ that approximates the US census values…".

Page 181, Exercise 5.19.f. The dilation factor is the square root of what is printed; it should be

$$\sqrt{\frac{(a+p)^2+(b+q)^2}{a^2+b^2}}$$

Page 181, Exercise 5.19.g. Replace the phrase "…deduce that $\theta > 0$ when $\mathbf{p} = (p,q)$ is above the line $q = (b/a)p$ and $\theta < 0$ below it." by "…deduce that $\theta > 0$ if and only if $\mathbf{p} = (p,q)$ is on the same side of the line $-bp+aq = 0$ as the vector $\mathbf{a}^{\perp} = (-b,a)$."

Page 182, Exercise 5.20.e. The domain of $\mathbf{s}$ needs to be restricted to the *interior* of W. Delete "$W \setminus (\pm\pi/2,0)$ (i.e., W with the two points $(\pm\pi/2,0)$ removed)" and replace it with "the interior of W, i.e., all points (x,y) with $-\pi/2 < x < \pi/2$".

Page 184, Exercise 5.25.b. The two appearances of $\mathbf{s}$ should be replaced by σ; thus "$\mathrm{d}\sigma_{(r,\mathbf{t})}$" and "$\det\left(\mathrm{d}\sigma_{(r,\mathbf{t})}\right)$".

Page 191, line +10. The last term in the displayed equation needs an additional parenthesis: "…$(x,g(x,f(x,y)))$".

Page 213, line +14. Replace "$dJ_{\mathbf{x}}$" by "$\mathrm{d}J_{\mathbf{x}}$".

Pages 216–217, Exercise 6.9.c. Adjust the range of u to "$1 \le u \le 2$", and correct the spelling of "images".

Page 244, line -7. The line should read "…matrix ML in a similar way…".

Page 246, line +6. Insert "…imaginary parts of the (eigenvalue) equation are…".

Page 247, line −9. The "little oh" should be in boldface: "…using $\boldsymbol{o}(s)/s \to 0\ldots$".

Page 258, line +13. A "Δ" is missing from the fourth expression in the displayed equation; the expression should read "$\ldots = \Delta\mathbf{x}^{\dagger}L^{\dagger}KL\Delta\mathbf{x} = \ldots$".

Page 264, lines +14 and +15. On each of these lines, replace "$d(\nabla f)_{\mathbf{a}}$" by "$\mathrm{d}(\nabla f)_{\mathbf{a}}$".

Page 265, Exercise 7.2 and Exercise 7.5. Correct the spelling of "matrix".

Page 267, Exercise 7.15. Springer style (see correction for page 104, above) requires the two occurrences of "e^x" be written as "e^x".

Page 267, Exercise 7.16. Correct the spelling of "written" in part (b) and "utility" in part (d).

page 295, title of Chapter 8.3. Correct the spelling of "Darboux" here, in the Table of Contents, and in the headings of the odd-numbered pages 295–311.

Page 313, Exercise 8.2.b. Correct the spelling of "analytically".

Page 314, Exercise 8.16. The numerator in the displayed expression needs a square root sign:

$$\frac{\sqrt{a^2+b^2+c^2+d^2 \pm 2(ab+cd)}}{\sqrt{2}}.$$

Page 315, Exercise 8.21. In part (c), add the following: "…at the point with polar coordinates $(a,b)\ldots$". In part (d), write the integral as

$$\iint\limits_{r\le\alpha} \rho(r,\theta)\,dA.$$

Page 330, First and second displayed equations. Replace the four occurrences of $1/m$ by $1/k$, thus:

$$I_k = \iint_{S_k} \frac{dA}{x} = \int_{-1}^{1}\int_{-1}^{-1/k} \frac{dx}{x}dy + \int_{-1}^{1}\int_{1/k}^{1} \frac{dx}{x}dy = 0,$$

and

$$\int_{-1}^{1}\int_{-1}^{-1/k} \frac{dx}{x}dy = -\int_{-1}^{1}\int_{1/k}^{1} \frac{dx}{x}dy.$$

Page 379, Exercise 9.9. Springer style requires the typographic change

$$\int_0^1\int_y^1 \mathrm{e}^{x^2}\,dx\,dy = \frac{\mathrm{e}-1}{2}.$$

Page 383, Exercise 9.32. In part (a), replace both occurrences of C by $\vec{C}$. In Part (c), make the typographic change

$$\oint_{\vec{C}} y\mathrm{e}^x\,dx + x\mathrm{e}^y\,dy.$$

Page 384, Exercise 9.38.d. The first integral should be $\iint_A \frac{du\,dv}{\sqrt{u^2+v^2}}$.

Page 386, Exercise 9.45.b. For clarity, add parentheses; thus $\ldots O((\Delta R)^2)$.

Page 409, Line +14. For clarity, the variables x, y, and z in the integral on the right should be replaced by ξ, η, and ζ, thus

$$\iint_{U^*} H(\mathbf{g}(s,t))\sqrt{\left[\frac{\partial(\eta,\zeta)}{\partial(s,t)}\right]^2 + \left[\frac{\partial(\zeta,\xi)}{\partial(s,t)}\right]^2 + \left[\frac{\partial(\xi,\eta)}{\partial(s,t)}\right]^2}\,ds\,dt.$$

Page 444, Exercise 10.6. Change the formula for y to $y = \frac{2v}{1+u^2+v^2}$.

Page 445, Exercise 10.11.g. Springer style requires the typographic change $(u\mathrm{e}^x - v\mathrm{e}^y)$.

Page 446, Exercise 10.15.b. To avoid confusion, rename $f(x)$ in the formula for ω, thus: $\omega(x,y) = h(x)\,dx + g(y)\,dy$.

Page 447, Exercise 10.23. Correct the area element in the third term of the integral:

$$\iint_{\vec{S}} xy\,dx\,dy + yz\,dy\,dz + zx\,dz\,dx.$$

Page 500, Line +1. Replace "1_i" by "i_1" in the multi-index $\widehat{I_s}$.

Page 511, Exercises 11.9 and 11.10. These are superfluous; they repeat Exercises 11.5 and 11.6.a.

Page 513, Exercise 11.23. Add "the" in "... the exterior derivative of the 0-form".