Agenda

1. Conditional Probability

In-Class Problems

- 1. Loaded dice. There are many ways to produce crooked dice. To *load* a die so that 6 comes up too often and 1 (which is opposite 6) comes up too seldom, add a bit of lead to the filling of the spot on the 1 face. Because the spot is solid plastic, this works even with transparent dice. If a die is loaded so that 6 comes up with probability 0.21 and the probabilities of the 2,3,4, and 5 faces are not affected, what is the assignment of probabilities to faces?
- 2. Is this calculation correct? Government data show that 6% of the American population are at least 75 years of age and that about 51% are women. Explain why it is wrong to conclude that because (0.06)(0.51) = 0.0306 about 3% of the population are women aged 75 or over.
- 3. Colored dice. Here's more evidence that our intuition about chance behavior is not very accurate. A six-sided die has four green and two red faces, all equally probable. Psychologists asked students to say which of these color sequences is most likely to come up at the beginning of a long set of rolls of this die:

RGRRR RGRRRG GRRRRR

More than 60% chose the second sequence. What is the correct probability of each sequence?

Warmup: Conditional Probability Suppose you have three cards: one is red on both sides, one is green on both sides, and the other is red on one side and green on the other. You draw a card and hold it up. You are looking at a red face. What is the probability that when you turn the card over, the other side will also be red?

In-Class Problems

- 1. Joint and conditional probabilities Pr(A) = 0.3, Pr(B) = 0.7
 - (a) Can you compute $Pr(A \cap B)$ if you only know Pr(A) and Pr(B)?

- (b) Assuming that events A and B arise from independent random processes,
 - i. what is $Pr(A \cap B)$?
 - ii. what is $Pr(A \cup B)$?
 - iii. what is Pr(A|B)?
- (c) If we are given that $Pr(A \cap B) = 0.1$, are the random variables giving rise to events A and B independent?
- (d) If we are given that $Pr(A \cap B) = 0.1$, what is Pr(A|B)?
- 2. Global warming A 2010 Pew Research poll asked 1,306 Americans "From what you've read and heard, is there solid evidence that the average temperature on earth has been getting warmer over the past few decades, or not?". The table below In the table below shows the distribution of responses by party and ideology, and the counts have been replaced with a relative frequencies.

		Response			
		Earth is	Not	Don't Know	
		warming	warming	Refuse	Total
Party and Ideology	Conservative Republican	0.11	0.20	0.02	0.33
	Mod/Lib Republican	0.06	0.06	0.01	0.13
	Mod/Cons Democrat	0.25	0.07	0.02	0.34
	Liberal Democrat	0.18	0.01	0.01	0.20
	Total	0.60	0.34	0.06	1

- (a) What is the probability that a randomly chosen respondent believes the earth is warming or is a liberal Democrat?
- (b) What is the probability that a randomly chosen respondent believes the earth is warming given that he is a liberal Democrat?
- (c) What is the probability that a randomly chosen respondent believes the earth is warming given that he is a conservative Republican?
- (d) Does it appear that whether or not a respondent believes the earth is warming is independent of their party and ideology? Explain your reasoning.
- (e) What is the probability that a randomly chosen respondent is a moderate/liberal Republican given that he does not believe that the earth is warming?
- 3. It's never lupus Lupus is a medical phenomenon where antibodies that are supposed to attack foreign cells to prevent infections instead see plasma proteins as foreign bodies, leading to a high risk of blood clotting. It is believed that 2% of the population suffer from this disease.

The test is 98% accurate if a person actually has the disease. The test is 74% accurate if a person does not have the disease.

There is a line from the Fox television show *House*, often used after a patient tests positive for lupus: "It's never lupus." Do you think there is truth to this statement? Use appropriate probabilities to support your answer.