List of Abstracts for Student Talks

Morning Talks

Links and link detection Gage Martin (Boston College) Live talk 11:00-11:15

Links are an important object of study in the field of low-dimensional topology. One way to describe a link is by drawing a picture of a link. This raises a natural question which is when are two different pictures really representing the same link? If we pick our favorite link, is there some set of steps we can follow to look at a picture and decide if it is a picture of our favorite link? In this talk we will mention some answers to these questions.

The exponential distance matrix of a graph Kate Lorenzen (Iowa State University) Pre-recorded talk 11:00-11:15

Given a graph G, the exponential distance matrix is defined entry-wise by letting the (u, v)-entry be $q^{dist(u,v)}$, where $dist(u, v)$ is the distance between the vertices u and v with the convention that if vertices are in different components, then $q^{dist(u,v)} = 0$. We establish several properties of the characteristic polynomial (spectrum) for this matrix and inertia of some graph families.

Why Cauchy Completeness and the Least Upper Bound Property are not Logically Equivalent Corrie Ingall (University of Connecticut) Live talk 11:00-11:15

There are many different ways to say that a metric space like the real numbers is complete. You could say that all Cauchy sequences converge. You could also say that all nonempty bounded sets have a least upper bound. The set of real numbers has both of these properties, so it may be tempting to say that both statements explain some universal notion of completeness. However, these properties are not logically equivalent and are in fact distinct. We will explore a counterexample that shows that Cauchy Completeness and the Least Upper Bound Property are not equivalent. This talk may be of particular interest to those who have taken an undergraduate course in Real Analysis, though anyone who has wondered why we need irrational numbers is welcome as well.

Adinkra Generating Algorithm and Program Felicia Flores (Bard College), Tobias Timofeyev (Bard College) Live talk 11:20-11:35

This 2020 Bard Summer Research Institute (BSRI) was led by Assistant Professor Mndez-Diez. The subject matter of our research was the Adinkra. This is a special type of graph meant to model supersymmetry. The goal of this BSRI project was to add to the functionality of a program started in a previous summer BSRI project. We would like this program, named adinkra-calc to output a graph of a true Adinkra, given only a couple parameters. The difficulty in this comes from the various properties an Adinkra has, and the different categories Adinkras can be sorted into.

Local Properties of Difference Sets Anqi Li (MIT) Live talk 11:20-11:35

Erdős and Shelah asked what we can learn about a large and complicated object X from properties that are satisfied by each small piece of X. We study the following variant of this problem, first studied by Erdős and Sós. Given a set of real numbers A, we consider the *difference set* $A - A = \{|a - b| : a, b \in A\}$. While a random set A is expected to have $|A - A| = \Theta(|A|^2)$, arithmetic progressions satisfy $|A - A| = \Theta(|A|)$.

Let $g(n, k, \ell)$ denote the minimum size of $|A-A|$, taken over all sets A of n numbers that satisfy the following local property: every subset $A' \subset A$ of k numbers satisfies $|A'-A'| \geq \ell$. Intuitively, every k numbers from A span many differences. We derive several new bounds for $g(n, k, \ell)$. We now state two of our results.

Erdős and others were interested in *linear thresholds* of local properties problems: the smallest ℓ for which the size of the global property is superlinear. We establish the linear threshold of the differences problem.

Theorem 1 *For every k, we have $g(n, k, k-1) = n - 1$ and $g(n, k, k) \gg n$.*

The *quadratic threshold* is the largest ℓ for which the size of the global property is sub-quadratic. We improve the current best bound for the quadratic threshold, although determining this threshold is still an open problem.

Theorem 2 *For sufficiently large k, we have*

$$g\left(n, k, \frac{17}{64}k^2 - \frac{5}{8}k + 7\right) = \Omega\left(n^2\right).$$

The following is the simplest of a family of bounds that we derive.

Theorem 3 *When k is a power of two, we have*

$$g\left(n, k, \frac{k^{\log_2(3)} + 1}{2}\right) = \Omega\left(n^{1+\frac{1}{k-1}}\right).$$

Triangles in non-Euclidean geometries: Elliptic and hyperbolic triangles and their areas Julia Starzycka (University of Illinois at Chicago) Live talk 11:20-11:35

Everyone is familiar with Euclidean geometry and can recite the formula for the area of the Euclidean triangle since elementary school. As a beginner-friendly introduction to other geometries, in this talk I will discuss how the properties of triangles are different depending on whether the triangle is considered in Euclidean, spherical, or hyperbolic geometry. The proofs for the areas in these geometries were very insightful to me and heightened my interest in mathematics and I hope my audience will be influenced in the same way.

I was introduced to this topic when reading "The Shape of Space" by Dr. Jeffrey Weeks. This book applies topology to cosmology and I recommend this book to anyone who is interested in learning new concepts after this talk.

Enumeration of Discrete Gradient Vector Fields on Simplicial Complexes Andrew Tawfeek (Amherst College) Live talk 11:40-11:55

Discrete Morse theory, developed over the past few decades since its original formulation by Forman in 1998, is a combinatorial analog to classical Morse theory. We provide a novel approach to enumerating the Forman equivalence classes of discrete Morse functions on finite simple graphs and explicitly show that their generating function is given by the characteristic polynomial of the graph Laplacian Δ – and illustrate what this connection says about acyclic matching occurring on the Hasse diagram of the graph. Furthermore, we provide a discussion of our current research on generalizing our results to Forman equivalence classes on higher-dimensional simplicial complexes – and what can be said about the generating function in larger dimensions.

Tverberg's Theorem, Disks and Hamiltonian Cycles Yaqian Tang (Wesleyan University) Live talk 11:40-11:55

Providing crucial insights to convexity, the Tverberg theorem is fundamental in the field of discrete geometry. The theorem states that $(r-1)(d+1)+1$ points in $\mathcal{R}^d$ can be partitioned into r subsets with intersecting convex hulls. In our project, we generalize the Tverberg type of partition for points to geometric graphs. In particular, we consider the family of disks whose diameters are the edges of the graph, and we are interested in which graphs induce a family of intersecting disks. Applying Helly's Theorem, we have shown that given an odd number of points in convex position on a plane, it is always possible to construct a Hamiltonian cycle with intersecting induced diametral disks.

Afternoon Talks

Fluid-Structure Interactions: Stability and error analysis for a loosely coupled scheme Rebecca Durst (Brown University) Live talk 2:00-2:15

We consider a fully discrete loosely coupled scheme for incompressible fluid-structure interaction based on the time semi-discrete splitting method introduced in [Burman, Durst & Guzman, arXiv:1911.06760]. The splittling method uses a Robin-Robin type coupling that allows for a partitioned solution of the solid and the fluid systems, without inner iterations. We prove that the method is unconditionally stable and robust with respect to the amount of added-mass in the system and provide an error estimate.

Generalizing Alder's conjecture on partitions with congruence relations and difference conditions Adriana Duncan (Tulane University), Simran Khunger (Carnegie Mellon University) Live talk 2:00-2:15

Integer partitions have long been of interest to number theorists, perhaps most notably Ramanujan, and are related to many areas of mathematics including combinatorics, modular forms, representation theory, analysis, and mathematical physics. Here, we focus on partitions with gap conditions and partitions with parts coming from fixed residue classes.

Let $\Delta_d^{(a,b)}(n) = q_d^{(a)}(n) - Q_d^{(b)}(n)$ where $q_d^{(a)}(n)$ counts the number of partitions of n into parts with difference at least d and size at least a, and $Q_d^{(b)}(n)$ counts the number of partitions into parts $\equiv \pm b \pmod{d+3}$. In 1956, Alder conjectured that $\Delta_d^{(1,1)}(n) \geq 0$ for all positive n and d. This conjecture was proved partially by Andrews in 1971, by Yee in 2008, and was fully resolved by Alfes, Jameson and Lemke Oliver in 2011. Alder's conjecture generalizes several well-known partition identities, including Euler's theorem that the number of partitions of n into odd parts equals the number of partitions of n into distinct parts, as well as the first of the famous Rogers-Ramanujan identities.

In 2020, Kang and Park constructed an extension of Alder's conjecture which relates to the second Rogers-Ramanujan identity by considering $\Delta_d^{(a,b,-)}(n) = q_d^{(a)}(n) - Q_d^{(b,-)}(n)$ where $Q_d^{(b,-)}(n)$ counts the number of partitions into parts $\equiv \pm b \pmod{d+3}$ excluding the $d+3-b$ part. Kang and Park conjectured that $\Delta_d^{(2,2,-)}(n) \geq 0$ for all $d \geq 1$ and $n \geq 0$, and proved this for $d = 2^r - 2$ and n even.

We prove Kang and Park's conjecture for all but finitely many d. Toward proving the remaining cases, we adapt work of Alfes, Jameson and Lemke Oliver to generate asymptotics for the related functions. Finally, we present a more generalized conjecture for higher $a = b$ and prove it for infinite classes of n and d.

Mathematical Analysis of a Model of Blood Flow Through a Channel with Flexible Walls Madeline Edwards (University of New Hampshire) Pre-recorded talk 2:00-2:15

Flutter is a naturally occurring and incredibly dangerous phenomenon. Flutter is the oscillation of a solid material, when exposed to specific conditions from fluid (liquid or gas) interaction, flows around and interacts with a rigid material, where the rigid material gains energy and the amplitude of oscillations of the structure increases. This amplitude increase is due to energy transferring from the fluid to the solid material. As the structure continues to interact with the fluid, the vibrations of the material increase without bound. It has been well studied that flutter can cause bridges to collapse, aircraft wings to break in mid-flight, and is a known cause of sleep apnea. Flutter is also naturally occurring in our blood vessels. This research concerns a long-standing bio-mathematical problem of deriving an analytical model to understand the flutter within blood vessels and the collapse of the membrane walls impacting the efficiency of blood flow. Here, we present the motivation and explicit solution to the initial boundary-value problem.

Posets and Interval Orders: an Approach to Resource Assignment Problems Aiyana Spear (Wellesley College), Abigail White (Wellesley College) Pre-recorded talk 2:00-2:15

What is the mathematics behind scheduling talks for a conference such as WIMIN? How do you optimize your time at a conference and schedule your day to attend the talks you are most interested in? In this talk we will introduce posets and interval orders, emphasizing how they are related to the problem of scheduling events. We will also explore the particular instance of all the events having the same length. Our hope is that you will leave this talk with a stronger understanding of posets and interval orders, convinced that a mathematical approach can help solve resource assignment problems.

A New Frobenius Template in a Matrix Ring Zhichun Zhang (Swarthmore College) Live talk 2:20-2:35

Frobenius problems, also known as Chicken Nugget problems, seek to find the largest nonnegative integer such that every integer after it can be written as a linear combination of the coprime generators using nonnegative integer coefficients. We now generalize Frobenius problems from a topic in number theory to a topic in ring theory. Previous to this work, research on generalized Frobenius problems has concentrated on the Gaussian integers and the rings Z[m], where m is a square-free positive integer. We launch the study of Frobenius problems in commutative rings of 2 2 upper triangular matrices with constant diagonal. Using properties of matrix rings and modular arithmetic, we determine for which lists of matrices with integer and real number coefficients the Frobenius set is non-empty. Additionally, for each list such that the Frobenius set is non-empty, we determine the range of the Frobenius set. For the lists of two matrices, we find the construction of every matrix in the Frobenius set. For the lists of more than two matrices, we find the conditions under which the construction can be extended. Finally, we introduce a new species of Frobenius problems for future studies

Operads and Localizations Emma Phillips (University of New Hampshire) Live talk 2:20-2:35

Given a category C and a collection of its morphisms, W, one can construct a localization of C with respect to W, which is a category where the morphisms in W are inverted. One can extend this notion of localization to operads, or collections of operations with a composition structure. We will introduce two constructions for a localization of an operad O with respect to a submonoid of its one-ary operations W, and present an ongoing analysis of a comparison functor which relates the two constructions. We believe that this comparison functor is a DK equivalence. This would imply that the newer construction, the tree hammock localization of operads, inherits the properties of the usual hammock localization, such as homotopy invariance and calculus of fractions. This is joint work with Maria Basterra.

The Adoption of M-Pesa: A Percolation Approach to Network Goods Lisa Reed (Union University), Janet Stefanov (Vanderbilt University), Zerrin Vural (University of Texas at Austin) Live talk 2:20-2:35

n 2007, Kenyas mobile network operator Safaricom launched M-Pesa, a mobile phone-based money transfer service. Today over 95% of Kenyan households use M-Pesa, making Kenya one of the first developing countries to fully embrace mobile payment systems. M-Pesa merits further academic investigation due to Kenya's resulting economic growth and poverty reduction since its initial launch. Here we reference percolation theory from statistical physics to develop a theoretical model of the spread of M-Pesa from 2007 to 2014. We consider M-Pesa to be a network good that spreads primarily via word of mouth and assume the adoption decision is determined by the utility a person can derive from it. This utility increases primarily with the number of M-Pesa users in one's social network. We simulate the spread of M-Pesa throughout Kenya using social network models and measure the goodness of fit of the model. Our model may be useful in analyzing the potential for the propagation of mobile money in other developing countries. We hope our findings will highlight the positive impact to be made by mobile money systems and motivate others to realize similar effects in developing countries.

Chains and Antichains in the Bipartite Cambrian and Tamari Lattices Rose Silver (Northeastern University) Pre-recorded talk 2:40-2:55

Dillworths Theorem states that the maximal size of an antichain is equal to the minimal number of chains needed to cover the partially ordered set. We study the Greene-Kleitman partition of c-Cambrian lattices of type A. We partially compute the Greene-Kleitman partition of the Bipartite Cambrian Lattice. We also study the Greene-Kleitman partition for the Tamari Lattice, a popular poset given by triangulations of polygons and the Catalan numbers. We give a lower bound for the size of a largest antichain for the Tamari Lattice.

This work started as an REU project at the University of Connecticut in Summer 2020. My mentor is Emily Gunawan, and the other students in the group are Ben Drucker, Eli Garcia, and Aubrey Rumbolt.

CIRCADA-I: Developing an interactive interface for chronobiological researchers
Amaya Smole (Amherst College) Pre-recorded talk 2:40-2:55

Chronobiology is a growing field that requires coordinated collaboration between chronobiologists and applied mathematicians. In order to address the potential gap in computational knowledge between these two groups involved in circadian rhythm data analysis, we created an interactive online Shiny app for researchers to process, denoise, and extract information from their data as desired. We developed our Shiny app under the guidance of a biomathematician at our institution and in collaboration with a chronobiological lab at a neighboring institution, implementing the use of the Lomb-Scargle periodogram, autocorrelation, cross-correlation, discrete wavelet transform, and sine-fitting analysis methods. Furthermore, our app allows users to process their data by removing outliers, detrending, denoising, and binning. Placing particular priority on accessibility, our implementation of thorough methods explanations, serif fonts, and helpful data visualizations make our analysis digestible for researchers of many backgrounds and abilities.

The ML degree of n-cycle models Xinyang Hu (University of Wisconsin) Pre-recorded talk 2:40-2:55

This project studies maximum likelihood estimation(MLE) from the viewpoint of algebraic geometry. The maximum likelihood (ML) degree, or the number of critical points of the likelihood function, is a widely studied topic. In the literature, Hosten, Ketan, and Sturmfels introduced the scaling coefficients for each statistical model, and asked the following question: How do scaling vectors change the ML degree? In this project, we show how to use scalings to reduce the ML degree of Binary-n- cycle models, a class of parametrized discrete exponentials encoded by nondecomposable undirected graphs. Geiger, Meek, Sturmfels stated that nondecomposable-undirected-graphs generally do NOT have a rational MLE. This leads to our main result: we use Kapranovs Horn Uniformisation to give a family of scaling vectors dropping the ML degree to 1 for arbitrary dimension, and we give the explicit form of the MLE as a rational polynomial. We will present a homotopy-tracking algorithm to find the MLE of generically scaled binary-n- cycle using our result. To illustrate the results, we will provide constructive proofs that the ML degree of scaled binary-4-cycle-models is bounded by 64, and some facial-unit scalings give ML degree exactly 1.