Lecture Notes - The Lever Rule

• The definition of the scale of a composition axis is generally chosen so that the values (e.g. mole fractions) vary from zero on the left to one on the right. It would be equally valid to redefine the axis so that it varied from zero on the right to one on the left. For example,

These two ways of showing the same information emphasize that the length of the lines on either side of the Mg$_2$SiO$_4$ point give the relative proportions of the components SiO$_2$ and MgO in the composition Mg$_2$SiO$_4$. The length of the line from the MgO point to the Mg$_2$SiO$_4$ point gives the (mass) proportion of SiO$_2$ in the composition Mg$_2$SiO$_4$. Similarly, the length of the line from the SiO$_2$ point to the Mg$_2$SiO$_4$ point gives the (mass) proportion of MgO in the composition Mg$_2$SiO$_4$. If weights of these proportions were placed on the appropriate ends of a seesaw or lever with the pivot point or fulcrum at the composition of Mg$_2$SiO$_4$, the lever would balance.

• The relationship between the length of connecting or tie lines and the proportions of compositions or minerals described above is a general one and is called the **lever rule**. The lever rule actually has a much broader application than simply reproducing information that might be read directly from a composition axis. For example, consider a hypothetical rock made of the minerals quartz (SiO$_2$) and forsterite (Mg$_2$SiO$_4$). Let the bulk composition of the rock be one that plots at the same point as MgSiO$_3$ on the composition axis. What are the relative mass proportions of quartz and forsterite in this rock? According to the lever rule,
the mass fractions are given by the lengths of the appropriate lines on the composition axis as follows:

<table>
<thead>
<tr>
<th></th>
<th>MgO</th>
<th>Mg$_2$SiO$_4$</th>
<th>MgSiO$_3$</th>
<th>SiO$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>.428</td>
<td>.600</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The mass proportion of forsterite is $(14/35)/(20/35) = 7/10$. The mass proportion of quartz is $(6/35)/(20/35) = 3/10$. This result may be confirmed by replotting the composition axis using SiO$_2$ and Mg$_2$SiO$_4$ as the defining end points. In this case, the composition of MgSiO$_3$ must be expressed as follows:

\[ 2 \text{ MgSiO}_3 = 1 \text{ Mg}_2\text{SiO}_4 + 1 \text{ SiO}_2 \]  
\[ 200 \text{ MgSiO}_3 = 140 \text{ Mg}_2\text{SiO}_4 + 60 \text{ SiO}_2 \]  
mole units  

and the resulting composition axis is:

\[ M_{\text{SiO}_2} = \frac{m_{\text{SiO}_2}}{m_{\text{Mg}_2\text{SiO}_4} + m_{\text{SiO}_2}} \]

Note that for this choice of "end point" compositions MgO plots at -0.75. Composition axes are not limited to values between zero and one. Also, points on a composition axis are not limited to compositions of real or even possible substances. The lever rule is useful for many questions that may arise when considering graphically the compositions of rocks and minerals. The lever rule works for any conservative unit of quantity (mass units, oxygen units, atom units, etc.). It will not always work for mole units!

- Two composition axes with one composition in common may be combined on the same two-dimensional graph to show two independent compositional variables. The result is a triangular or ternary composition diagram. The attached ternary diagram is contoured (sequentially) in terms of the three plotting coordinates \([B/(A+B+C)], [C/(A+B+C)], \) and \([A/(A+B+C)]\). Note that only two of the three coordinates are independent. Constant values of the ratios A/B, B/C, and C/A are also shown for a ternary diagram. Observe that the choice of an equilateral triangle is arbitrary. The same plotting coordinates may be used for other shapes of triangles. When the compositions of three phases (e.g. minerals or melts) in equilibrium are at the corners of the triangle, the proportions of those phases are given by the values of the three plotting coordinates (the ternary lever rule) if a conservative unit of quantity is being used.