A Crustal Geostatic Gradient

Pressure increases with depth in the earth due to the increasing mass of the rock overburden. Computing the pressure as a function of depth in a homogeneous crust is a straightforward calculation. In SI units, pressure (**Pa**scals) is the force (**N**ewtons) per unit area (**m**eters^2) such that

$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

You may also see pressure written as **bars** or **atm**ospheres with

$$1 \text{ bar} = 1 \times 10^5 \text{ Pa} = 0.9872 \text{ atm}.$$

To see how the pressure would increase with depth in the crust (the geostatic gradient), consider the pressure beneath a one meter cube of granite (density = $2.8\text{x}10^3$ kg/m^3). The force applied by the $2.8\text{x}10^3$ kg of this cube to the rocks beneath it is given by

$$\text{force} = \text{mass x acceleration} = (2.8\text{x}10^3)\ (9.8 \text{ m/s}^2) = 2.7\text{x}10^4 \text{ N}.$$

where (9.8 m/s^2) = **g**, the acceleration of gravity at the surface of the earth. Because this force is distributed across the 1 m^2 area of the base of the cube, the pressure beneath the cube is

$$\text{pressure} = \frac{2.7\text{x}10^4 \text{ N}}{1 \text{ m}^2} = 2.7\text{x}10^4 \text{ Pa}.$$

If another cube is placed on top of the first one, the pressure under the two cubes will be $5.4\text{x}10^4$ Pa. As more cubes are stacked, the pressure at the base rises at the rate of

$$2.7\text{x}10^4 \text{ Pa/m} = 2.7\text{x}10^7 \text{ Pa/km} = 27 \text{ MPa/km} = 270 \text{ bars/km}$$

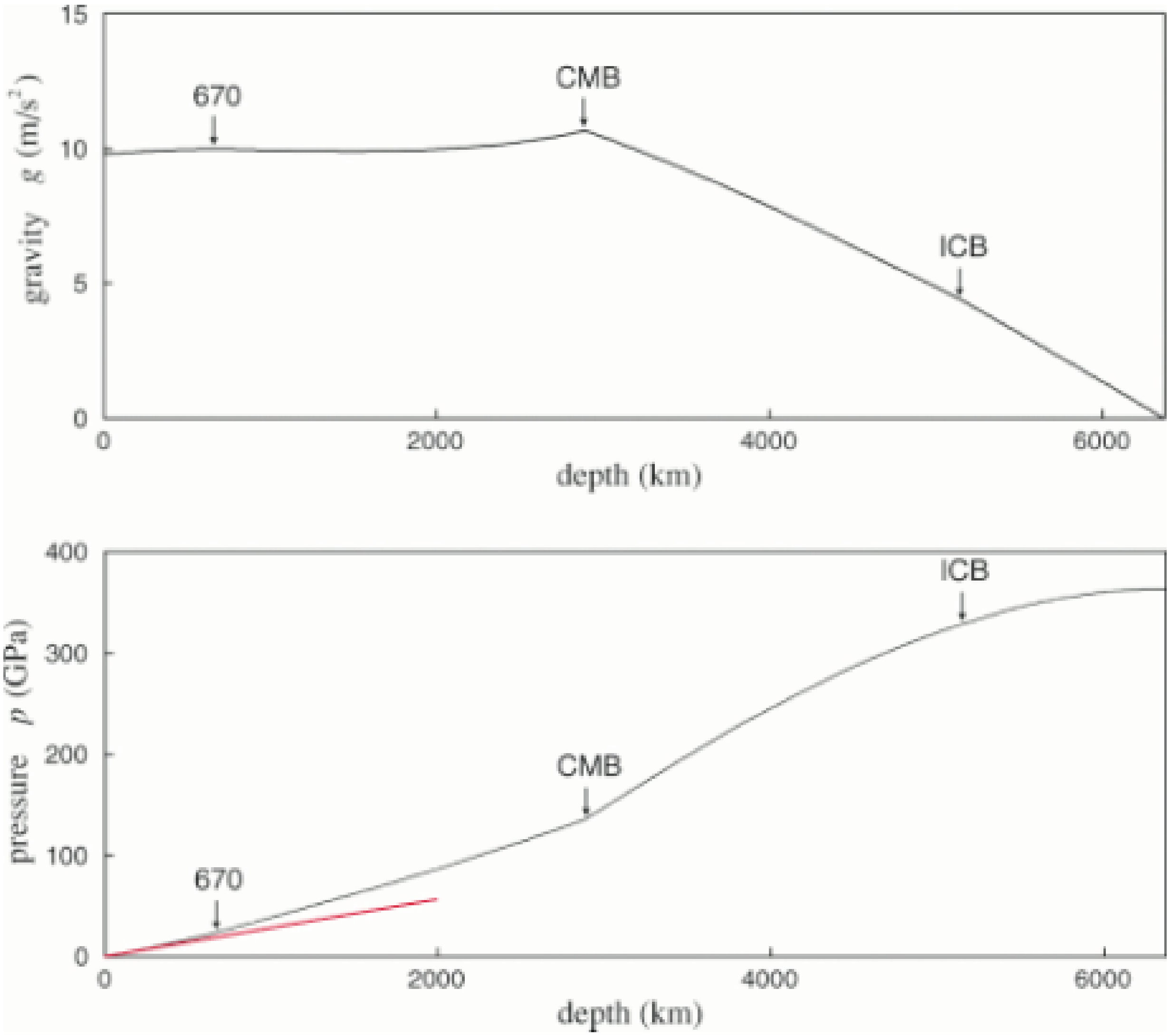
where MPa (=10^6 Pa) stands for megapascals. Alternatively, this pressure distribution may be expressed as

$$3.7 \text{ km/kbar} = 37 \text{ km/GPa}$$

where GPa (=10^9 Pa) stands for gigapascals. Remember that these numbers are only correct for a uniform crustal density of $2.8\text{x}10^3$ kg/m^3. Higher densities will yield higher pressure gradients. The geostatic gradient changes with depth as the density increases. Our procedure may be generalized to the earth with the following differential equation:

$$\frac{dP(r)}{dr} = -g(r)\rho(r)$$

where r is the radial distance from the center of the earth. By integrating this equation, pressure can be found for any depth if density and gravity are known. Density, gravity, and therefore pressure vary with depth as shown in the following graphs found in Tromp (2001):



Our linear approximation predicts a pressure of 54 GPa at a depth of 2000 km, whereas the model shown in the graph predicts a pressure of 87 GPa at that depth.