

Enthalpy of Magmatic Crystallization

In the 1997 movie, *Volcano*, a basaltic volcano erupts in the heart of Los Angeles. Pahoehoe lava pours down the streets, destroying buildings, starting fires, and causing general chaos. In one of the climactic scenes of the film, Tommy Lee Jones organizes firefighters, public works employees, citizens, and National Guard helicopters to stop the main lava flow. They do this in a matter of minutes by blocking the flow with concrete traffic barriers, then chilling the lava with water pumped by firehoses and dropped from helicopters.

As a geologist with knowledge of lava, you might be asked to assess the probability of success of a defense against lava of this kind. Based on the information given below about lava and water,

- (1) Estimate how much water would have to be poured on the lava to stop the flow.
- (2) Calculate how long it would take for the firefighters to pump this much water.
- (3) Calculate how many helicopter drops it would take to drop this much water.
- (4) Make a list of the additional assumptions you make for your calculation.
- (5) Be sure to consider uncertainties in your calculation and use only significant figures in your answer.

Assume that:

- 1) The lava flow is confined between buildings 30 m apart so that you must cool an area of 30 m x 150 m.
- 2) The lava flow needs to solidify to a depth of 1 meter over the surface area of 4,500 m² in order to create a crust sufficiently strong and extensive to resist the pressure from upstream of the still molten and flowing magma.
- 3) Once the water sprayed on the magma has turned to steam, it escapes, so that it cannot further absorb heat from the magma.
- 4) Densities of magma and solid basalt are approximately the same. (They actually differ by about 10%, but this calculation is only an estimate so the 10% will not matter much.)

Mathematical Relationships

Mass (kg) = density (g m⁻³) x volume (m³)

Energy transferred by changing state (J) = latent heat (J kg⁻¹) x mass (kg)

Energy transferred to change the temperature (J)

= specific heat (J kg⁻¹K⁻¹) x temperature change (K) x mass (kg)

Numbers that you may find useful

Density of basalt or basaltic magma: 3.0 x 10³ kg m⁻³

Specific heat of basaltic magma: 1.0 x 10³ J kg⁻¹ °C⁻¹

Specific heat of (solid) basalt: 1.4 x 10³ J kg⁻¹ °C⁻¹

Latent heat of crystallization of basaltic magma: 4.0 x 10⁵ J kg⁻¹

Temperature of the magma: 1350°C

Crystallization temperature of basalt: 1100°C

Density of water: $1.0 \times 10^3 \text{ kg m}^{-3}$

Specific heat of steam: $2.0 \times 10^3 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$

Specific heat of liquid water: $4.2 \times 10^3 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$

Latent heat of vaporization of water: $2.3 \times 10^6 \text{ J kg}^{-1}$

Temperature of water when sprayed: 20°C

Carrying capacity of a fire truck (internal tanks): 2000 gallons

Carrying capacity of a helicopter: 500 gallons

Pumping rate of a fire truck (if water supplied from city water main via fire hydrants): 1500 gallons/min

Number of fire trucks: 20

Number of helicopters: 20

Equivalencies

1 erg = 10^{-7} J

1 gallon of water = 3.8 liters = 3.8 kg

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