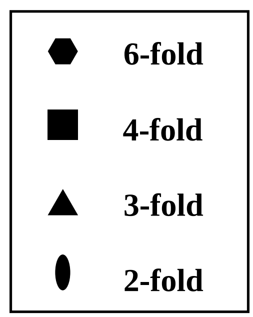
Lecture Notes - Mineralogy - Stereographic Projections

- The **stereographic projection** is a device use by mineralogists and structural geologists to represent 3-dimensional information in two dimensions. Mineralogists use a **Wulff** stereonet, which is constructed from a simple geometric recipe. Structural geologists use a **Schmidt** stereonet, which is constructed from a more complicated algorithm designed so that each “square” on the net has the same area.

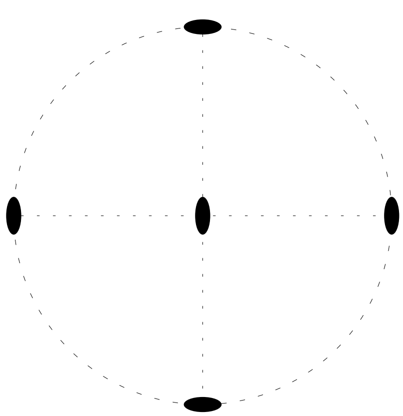
- The Wulff sterographic projection is constructed by projecting points and lines from the surface of a sphere onto a horizontal plane passing through the center of the sphere. The projection is accomplished by connecting the bottom point of the sphere (the south pole) with the points on the upper (northern) hemisphere. Each point from the upper hemisphere plots where the line of projection passes through the plane of projection (the equitorial plane). Points on the bottom (southern) hemisphere can be projected “negatively” using the upper (north) pole as the point of projection. (Structural geologists normally represent the bottom hemisphere on their Schmidt stereographic projections.)

- The **stereonet** itself shows the projection of great circles and small circles. A **great circle** is the line of intersection with the surface of a sphere of a plane that passes through the center of the sphere. A **small circle** marks the path in space of a point on the surface of a sphere that is rotating about an axis. Lines of longitude on the earth are great circles; lines of latitude are small circles. These reference lines are useful in the manipulation of crystallographic data.

- All of the symmetry elements of a crystal class and their relative positions may be shown conveniently on a sterographic projection. The symmetry point of the point group is placed in the center of the sphere of projection. Intersections with the upper hemisphere of the symmetry elements of the point group (rotation axes, rotoinversion axes, and/or mirror planes) are projected onto the stereonet plane. By convention (Klein and Hurlbut, p.62), the c-axis is chosen as the vertical axis, the b-axis is east-west, and the a-axis is north-south. Please follow this convention, but be aware that not all authors follow it.



- **Rotation axes** are lines and, therefore, intersect the upper hemisphere as points. The stereographic projections of these points are represented by filled "polygons" with the same number of sides as the "fold" of the axis. If the rotation axis is vertical or inclined, there will be only one intersection on the upper hemisphere and, therefore, only one polygon on the projection. However, if the rotation axis is horizontal, both ends of the axis will intersect the upper hemisphere and, therefore, two polygons will plot on the projection on opposide sides of the perimeter of the projection. An open circle in the center of the polygon (or alone in the center of the projection) signifies the presence of a **center of symmetry**. Four-fold rotoinversion axes are signified by an open 2-fold symbol inside a filled square.



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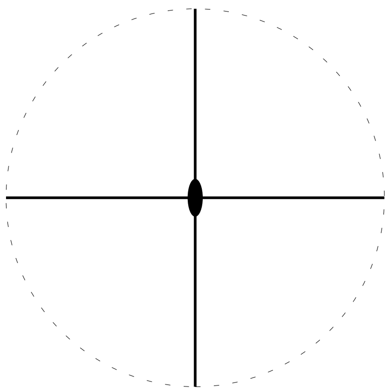
- Mirror planes intersect the upper hemisphere in great circles. In all but two of the crystal classes the mirror planes are either horizontal or vertical. A horizontal mirror plane projects as the circle that is the perimeter of the stereographic projection. A vertical mirror projects as a straight line. Only two crystal classes have inclined mirrors and these mirrors are at 45°.



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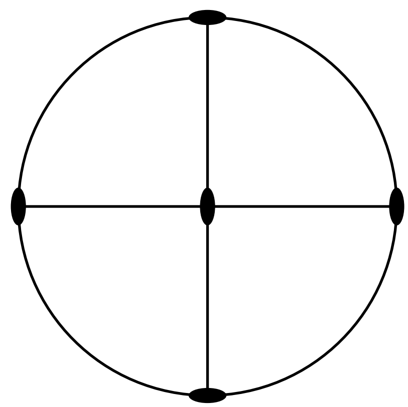

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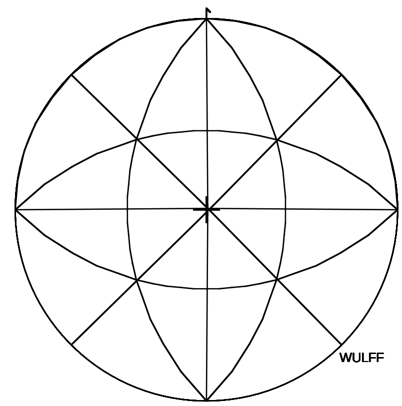

- Only certain combinations of symmetry elements are possible and, for the same reasons, some symmetry elements require the presence of others. Particularly useful to remember is that **the line of intersection of any two mirror planes must be a rotation axis**. If the mirror planes intersect at 90°, the line of intersection will be a 2-fold axis; at 60°=> 3-fold; at 45°=> 4-fold; at 30°=> 6-fold.

- Identification of the crystal class of crystals (or models of crystals) is facilitated by a chart giving stereographic projections of the symmetry elements of each crystal class. The nature and relative position of every symmetry element identified can be used to eliminate many possible classes. Reference to the stereographic projections of remaining possibilities will show where to look for additional confirming symmetry elements.

- Crystal faces may be represented on a stereographic projection by their poles. Imagine a tiny version of the crystal located with its center at the center of the sphere of projection. For each face extend a line perpendicular to the face (it's **pole**) from the crystal out through the sphere. The point of intersection of the pole with the sphere is then projected stereographically onto the horizontal plane. If the pole intersects the upper hemisphere, it is shown as a solid point on the projection. If the pole intersects the lower hemisphere, it is shown as a tiny open circle.

- In ideal crystals, each face is repeated by all of the symmetry elements of the point group of that crystal. For example, if the face (110) is present on a tetragonal crystal, the three faces symmetrically equivalent to (110) by the 4-fold axis should also be present. These are ($1\bar{1}0$), ($\bar{1}10$), and ($\bar{1}\bar{1}0$). The set of crystal faces in a crystal class that are symmetrically equivalent are called a **form** for that class and are designated by braces around the Miller indices of one of the faces. For example, the four faces mentioned above may be represented by the one symbol {110}. This form is called a **tetragonal prism** (see Table 2.5 of Klein and Hurlbut). All of the common forms are shown in Figure 2.35 of Klein and Hurlbut. Note that an individual crystal may have a **habit** or shape that includes several forms.