Lecture Notes - Optics 1: Electromagnetic Waves, Snell's Law

• We now embark on a four-week “digression” to consider the physics of the interaction of light with non-opaque minerals. Application of these principles through the use of a polarizing microscope allows geologists to confirm the hand specimen identification of minerals, to identify mysterious unknown samples, to examine the microscopic textural information contained in rocks, etc.

• Although light has properties in common with both waves and particles, the wave theory is most useful in understanding the interaction of polarized light with crystals. Light is the visible form of electromagnetic radiation. Each light wave consists of propagating electric and magnetic fields that may be characterized by electric and magnetic vectors oriented perpendicular to each other and to the direction of propagation in isotropic substances. Because the electrical interaction of light with crystals is much stronger than the magnetic interaction, we will focus on the electric vector.

• As a light wave passes a point, the electric and magnetic vectors at that point grow and shrink as the electric and magnetic fields rise and fall (see Nesse, Figure 1.2). Plotting the magnitude of the electric vector with time as a light wave passes, one obtains a graph of a sine function. The direction indicated by the rising and falling electric vector as light passes is called the vibration direction for the light wave of interest. The distance between wave crests is the wavelength ($\lambda$), which for light in a vacuum is between about 400 and 700 nm. The frequency ($f$) of vibration for light is about 10$^{15}$ hertz. The wavelength $\lambda$ (m), frequency $f$ (s$^{-1}$), and veolcity $v$ (m/s) of an electromagnetic wave are related by the expression: $v = \lambda f$.

• Two light waves can interfere with each other to produce a resultant wave. If the two waves are in phase, the resultant wave will have a greater amplitude. However, if the two waves are out of phase by a value near $\lambda/2$, the resultant wave will have a much reduced amplitude. Here are two examples with the resultant wave shown below the two interfering waves:
• Light travels fastest in a vacuum where its velocity is $3 \times 10^8$ m/s. In other substances (notably minerals!) the velocity of light is less than $3 \times 10^8$ m/s. The ratio of the velocity of light in a vacuum to the velocity of light in another medium is called the **refractive index** of the medium. Refractive indices are always greater than one. Refractive indices for minerals are typically between 1.4 and 2.0. The refractive index of water (at 25°C) is 1.33. The refractive index of air (at 25°C and 1 bar) is 1.0003. Because the frequency of light is the same in all media, the wavelength of light must be shorter when its velocity is lower ($v = \lambda f$).

• When a ray of light passes from one medium to another, the propagation direction generally changes. This process, called **refraction**, is the principle behind lenses and may be used to explain many common observations from “bent oars” to internal reflection in prisms. The degree of refraction or bending depends on the relative refractive indices of the two mediums according to a relationship called **Snell's Law**

\[
\sin \theta_1 \frac{\sin \theta_2}{\sin \theta_1} = \frac{n_2}{n_1}
\]

where $n_1$ and $n_2$ are the refractive indices of the two media and $\theta_1$ and $\theta_2$ are the angles of incidence and refraction. Note that the angles relate the ray path to the line that is perpendicular to the interface. Note also that the same relation applies whichever direction the light is traveling. Snell's Law is named for Willebrod Snel (1580-1626), a Dutch mathematician and astronomer who described this relationship in 1621. (Snel was married in 1608 and had 18 children, of whom only 3 survived.)
• Because the sine function cannot have a value greater than one, when \( n_1 < n_2 \) there may be no value of \( \theta_1 \) that will satisfy the expression

\[
\sin \theta_1 = \sin \theta_2 \frac{n_2}{n_1}
\]

for some values of \( \theta_2 \). Thus, for light passing from a medium with a higher refractive index to a medium with a lower refractive index there is a **critical angle** of incidence, above which there is **total reflection** rather than refraction. This feature of refraction is used to construct **refractometers** to measure the refractive indices of liquids and large crystals.