Introduction
To Recursive Functions

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LARGE TASK
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AFTER A WHILE...
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THE ANSWER!
Important Concept: 1

• Recursive step reduces the problem in a small, but significant way, getting closer to a solution

• Work done during a recursive call builds up on the partial solution found so far.
Important Concept: 2

- Recursion requires
  
  1. **Stopping** Condition

  2. Recursive Step **Reducing Size of Problem** and leading closer to solution.
Examples
Draft Algorithm, then Code

• Factorial

• Sum up an array

• Find the largest element of an array

• Find a key in an unsorted array

• Find a key in a sorted array

• Evaluate an RPN expression
Evaluating Time Complexity

- Factorial

```java
private static int factorial(int n) {
    if (n <= 1) {
        return 1;
    }
    return n * factorial(n - 1);
}
```
Evaluating Time Complexity

• Binary Search

```java
private static int binSearch( ArrayList A, int low, int high, int key ) {
    if ( low > high )
        return -1;

    int mid = ( low+high )/2;
    if ( (int) A.get( mid ) == key )
        return mid;

    if ( (int) A.get( mid ) < key )
        return binSearch( A, mid+1, high, key );
    else
        return binSearch( A, low, mid-1, key );
}
```
Tail Recursion
Let's Revisit Fibonacci

```java
private static long computeFibRecursively(int n) {
    if (n <= 1)
        return 1;
    return computeFibRecursively(n - 1) + computeFibRecursively(n - 2);
}
```
Fib's Call Tree

Observations:

• So Many leaves!

• Most of the work in the lower part of the tree, where the leaves are...

• If we could "prune" the tree, we could reduce the amount of work done...
Solution? Cut the Tail-End Recursion!

```java
private static long computeFibRecursively( int n ) {
    long[] f10 = new long[] {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89};

    if ( n<= 10 )
        return f10[n];

    return computeFibRecursively( n-1 )
        + computeFibRecursively( n-2 );
}
```
[beowulf2]
[07:07:35] ~/public_html/classes/212$:

[beowulf2]
[07:07:36] ~/public_html/classes/212$:
[beowulf2]
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[07:07:36] ~/public_html/classes/212$:

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[07:07:38] ~/public_html/classes/212$:
The N-Queens Problem
Question: Can one put 8 queens on a chess board, such that no two queens can take each other?
Questions Before Coding

- What data structure can we use?
- How do we represent a placed queen?
- How do we represent a cell "covered" by a queen?
- How do we represent an empty cell?
Important Concept Of the Day

• **Back-Tracking**: the action of returning from recursive exploration of a sub-problem, undoing some computation, selecting a new unexplored path, and starting exploring it recursively.