Objective of the course

The purpose of this course is to review mathematical methods commonly used in economic theory. It will cover concepts and techniques of multivariate calculus and linear algebra and applications to analyze problems arise in economics. It is designed to give first-year graduate students some "survival" skills to deal with the fundamental problems appeared in graduate economic courses.

Textbooks

Main textbook

Additional reading
Course outline

*Topic 0: Introduction*

0.1. Mathematics as the "language" of economics
0.2. Optimization problems in economic theory: models of consumer choice and models of producers choice

*Topic 1: Linear algebra*

1.1. Basic topological concepts: element, sequences, and set
1.2. Linear vector space and linear algebra: vectors, scalar product, linear combinations, linear independence, system of linear equations, hyperplanes, matrices, operations over matrices, rank of a matrix and linear independence, norm of a vector, convex set.

*Topic 2: Calculus of several variables*

2.1. Functions of single variable: continuity, differentiation, Taylor formula, Mean Value Theorem, extrema
2.2. Functions of several variables: chain rule for differentiation of composite functions of several variables, gradient vector, directional derivative, total differential, level curves and surfaces, Jacobian matrix, implicit function theorem, convex and concave functions.

*Topic 3: Optimization in economic theory and comparative statics analysis*

3.1. Unconstrained optimization: optimization criteria
3.2. Constrained optimization: optimization subject to equality constraints (Lagrange’s principle); optimization subject to inequality constraints; examples of consumer’ choice problems and producer’s choice problems; optimization subject to inequality constraints and non-negativity constraints (Kuhn-Tucker Theorem)
3.3. Comparative statics: consumer’ choice problems and producer’s choice problems

*Topic 4: Dynamic optimization*

4.1. Introduction to dynamic optimization
4.2. Dynamic Programing Method. Solving finite and infinite horizon dynamic programming problems, Bellman’s equation: value function as a fixed point of mapping.
4.3. Contraction Mapping Theorem
4.4. Stability of dynamic systems and their linearization. System of first order linear difference equations


**Topic 5: Dynamic systems and analysis**

5.1. Introduction to a system of difference equations: one-dimension linear difference equations; two-dimension first-order difference equations; solution methods; second-order or $k$th-order difference equations

5.2. Solution methods: eigenvalue, eigenvector, matrix decompositions

5.3. Dynamic systems of neoclassical growth model and overlapping generation model.

**Topic 6: Numerical analysis in economics (using Matlab)**

6.1. Basics of Matlab

6.2. Linear Algebra with Matlab

6.3. Optimization with Matlab

6.4. Solving simple economic problems with your owned-Matlab functions
Course plan

Session 1: Introduction and Linear algebra
- Reading: Chapters 6, 7, 8, 9 and 10 (Simon and Blume)
- Linear vector space and linear algebra
- Basic topological concepts

Session 2: Calculus of several variables
- Reading: Chapters 2, 3, 13, 14 and 15 (Simon and Blume)
- Foundations of functions of a single variable: Limit, continuity, derivative. Taylor formula for functions of a single variable. First and second order conditions of convexity and concavity for functions of a single variable
- Foundations of functions of several variables: Gradient vector, directional derivative, total differential, level curves and surfaces, Jacobian matrix, implicit function theorem, convex and concave functions, Taylor formula for functions of several variable. First and second order conditions of convexity and concavity for functions of several variable
- Linear algebra with Matlab

Session 3: Optimization in economic theory and comparative statics
- Reading: Chapters 16, 17, 18 and 19 (Simon and Blume)
- Introduction to optimization in economic theory: static and intertemporal utility maximization problem; Firm’s choice: profit maximization problems, cost minimization problems
- Optimization without constraint
- Optimization with equality and inequality constraints (Lagrange Principle)
- Optimization with inequality constraints and non-negativity constraints (Kuhn-Tucker Lagrange Principle)
- Comparative statics
- Optimization toolbox in Matlab
Session 4: Dynamic optimization and dynamic analysis

• Reading: Chapters 23 (Simon and Blume)

• Introduction to dynamic optimization

• System of difference equations: one-dimension linear difference equations; two-dimension first-order difference equations; solution methods (eigenvalue, eigenvector, matrix decompositions); second-order or \( k \)-th-order difference equations;

• Application to neoclassical growth model and overlapping generation models.

• A system of difference equations in Matlab

Session 5: Numerical analysis using Matlab

• Introduction to programming in Matlab

• Examples of solving consumer problems, producer problems and a simple general equilibrium model