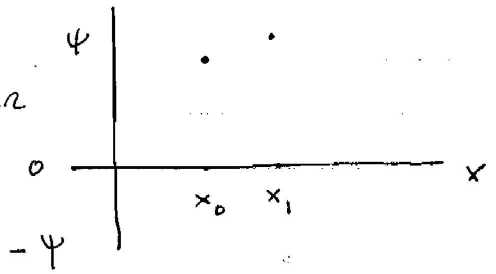
21 SEP '09

CONSIDER



$\psi'(x_0) > 0 \Rightarrow \psi_{x_1} > \psi_{x_0}$

AS $\psi_1 \approx \psi_0 + \psi'(x_0)(x_1 - x_0)$

(ELEMENTARY CALCULUS)

NOW THROW IN THE (REARRANGED) SCHRÖDINGER EQUATION

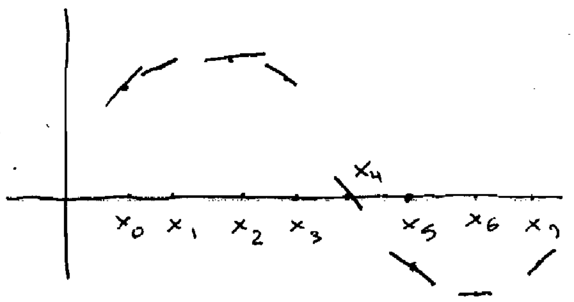
$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2m}{\hbar}(E - V)\psi$$

WE ARE ASSUMING $\psi > 0$ ++ WE KNOW $m, \hbar > 0$

FURTHER ASSUME $E - V > 0 \;\forall\; x$

S.E $\Rightarrow$ SIGN $\frac{\partial^2 \psi}{\partial x^2}$ = $-$ SIGN ψ

SO AT x_0 $\psi'(x)$ IS DECREASING AS $\psi > 0$ (ASSUMPTION)
THIS MEANS AS $x \uparrow$ $\psi'(x) \downarrow$ SO IT WILL TURN TO 0 AT x_2 ++ NEGATIVE AT x_3 THIS $\Rightarrow$ $\psi(x)$ WILL HIT 0 AT x_4 ++ BE NEGATIVE AT x_5



AS SOON AS $\psi(x) < 0$ $\frac{\partial^2 \psi}{\partial x} > 0$ WHICH MEANS $\psi'(x)$ WILL $\uparrow$ AS $x \uparrow$

SO x WILL START HEADING BACK TO 0

THE CYCLE REPEATS ++ WE GET OSCILLATORY BEHAVIOR OF $\psi(x)$ AS $x \uparrow$

	$\psi(x)$	$\psi'(x)$
x_0	> 0	> 0
x_3	> 0	< 0
x_5	< 0	< 0
x_7	< 0	> 0

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NEXT CONSIDER THE SAME SITUATION BUT WITH $(E-V) < 0$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-V)\psi \Longrightarrow \text{SIGN}\,\frac{\partial^2 \psi}{\partial x^2} = \text{SIGN}\,\psi$$

IT'S CLEAR $\psi > 0$ & $\frac{\partial \psi}{\partial x} > 0 \Longrightarrow \psi$ BLOWS UP AS $\uparrow$

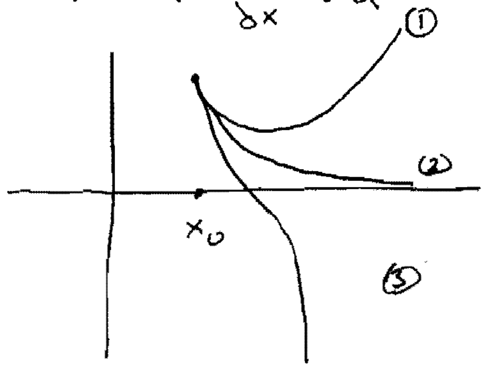
SIMILARLY

$\psi < 0$ & $\frac{\partial \psi}{\partial x} < 0 \Longrightarrow \psi \to -\infty$ (BLOWS UP) AS $x \uparrow$

THE REALLY INTERESTING CASE IS

$\psi > 0$ $\frac{\partial \psi}{\partial x} < 0$ RECALL THAT $\psi > 0$, $\frac{\partial^2 \psi}{\partial x^2} > 0 \Longrightarrow \frac{\partial \psi}{\partial x} \uparrow$ AS $x \uparrow$

$\exists$ 3 POSSIBILITIES START AT x_0



(1) ψ NEVER GETS TO 0 BEFORE $\frac{\partial^2 \psi}{\partial x^2}$ TURNS IT AROUND & ψ BLOWS UP

(3) ψ GOES BELOW 0 & $\frac{\partial^2 \psi}{\partial x^2}$ DOES TOO MAKING $\psi'(x)$ EVEN MORE NEGATIVE & BLOWING IT UP

(2) ψ APPROACHES 0 AS $\psi > 0$ & $\frac{\partial^2 \psi}{\partial x^2} > 0$ & $\psi'(x)$ STAYS NEGATIVE

SINCE $V(x)$ IS ASSUMED CONSTANT (TIME INDEPENDENT S.E)
THE 3 POSSIBILITIES ARE DETERMINED BY E

IT'S MUCH MORE COMPLICATED IF BOTH E & V CAN VARY WITH X.