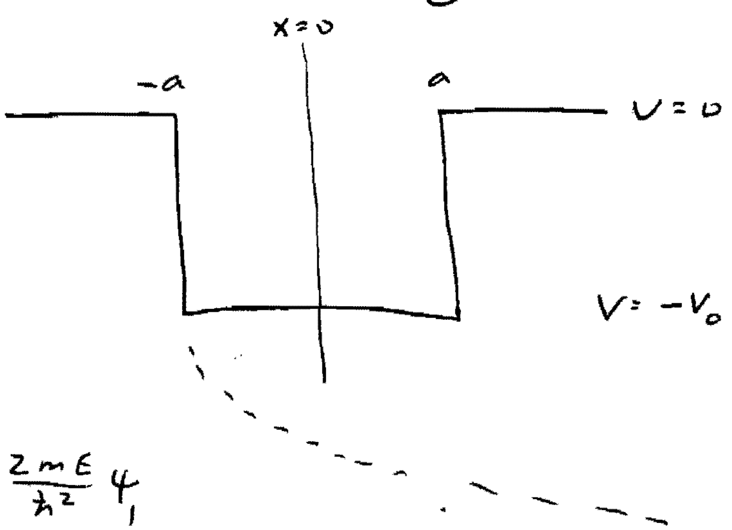
Fn



V_0 a positive #

S.E.

$$\frac{\partial^2}{\partial x^2}\psi_1 = -\frac{2mE}{\hbar^2}\psi_1$$

E is negative, so to get a <u>real</u> k I will set $\left(-\frac{2mE}{\hbar^2}\right) = k^2$

$$k = \sqrt{-\frac{2mE}{\hbar^2}}$$

Then

$\psi_1 = Ae^{kx} + Be^{-kx}$ satisfies S.E. (prove it if you doubt)

at large $-x$ value, $e^{-k(-x)}$ blows up so $B \to 0$

$$\psi_1 = Ae^{kx}$$

in middle region, guess

$$\psi_2 = C\sin(lx) + D\cos(lx)$$

where S.E. says

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_2 + V\psi_2 = E\psi_2$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_2 = (E - V)\psi_2$$

$$\frac{\partial^2}{\partial x^2}\psi_2 = -\frac{2m}{\hbar^2}(E - V)\psi_2$$

$$= -\frac{2m}{\hbar^2}(E + V_0)\psi_2$$ since $V = -V_0$

Now $\underbrace{-\frac{2m}{\hbar^2}\overbrace{(E+V_0)}^{\text{pos since } E = \text{neg, but } |E|<|V_0| \text{, } V_0 \text{ positive}}}_{\text{pos, so set it } = \ell^2}$

$$\frac{\partial^2 \psi_2}{\partial x^2} = -\ell^2 \psi_2$$ & the C/D eqtn is a solution.

If we take only symmetrical answer,

$C \to 0$ since $\sin[\ell x]$ is asymmetrical.

So boundary conditions are:

$\psi_1(x=-a) = \psi_2(x=-a)$ or $Ae^{-ka} = D\cos(-\ell a)$

$Ae^{-ka} = D\cos(\ell a)$

and $\frac{\partial \psi_1}{\partial x}(x=-a) = \frac{\partial \psi_2}{\partial x}(x=-a)$

$+kAe^{+kx}\Big|_{x=-a} = -\ell D\sin(\ell x)\Big|_{x=-a}$

$kAe^{-ka} = -\ell D\sin(-\ell a) = \ell D\sin(\ell a)$

$k\cancel{D}\cos(\ell a) = \ell\cancel{D}\sin(\ell a)$

$$\frac{k}{\ell} = \frac{\sin(\ell a)}{\cos(\ell a)} = \tan(\ell a)$$

$$k = \sqrt{\frac{-2mE}{\hbar^2}}$$ & with $m=1$, $\hbar=1$, this term

$$k = \sqrt{-2\cdot(-e)}$$ e is what I use in M.

$$\ell = \sqrt{\frac{2m}{\hbar^2}(E+V_0)}$$

$$= \sqrt{2(e+100)}$$

See mathematica page on transcendental functions.