Smith College
Chemistry 331, Fall 2011
Mr. Linck

Exercises, Set One

1.

1.1. Calculate the energy emitted per second from a 60 watt light bulb. HINT: A watt is a joule/sec.

1.2. Assume 0.30 of that energy is visible light, which you can assume (for ease of calculation) has a λ of 500 nm. That energy is emitted in all directions and the ladybug of interest to us occupies only a part of all those directions. Assume she is 1.0 meters from the source. What fraction of the spherical surface does she occupy?

1.3. How many 500 nm photons strike the ladybug in a second, the second it takes you to observe her, assuming all the photons of interest are at 500 nm (see 1.2.)?

1.4. We now work with conservation of momentum. The Einstein relationship gives the momentum of a photon as h/λ. So the total momentum incident on the ladybug in a second is (Number of photons)(momentum per photon). This is the momentum transfered to the ladybug. What is her velocity due to being hit by the light?

1.5. Is the ladybug large in the Dirac sense?

2.

If you have plane polarized light, say vertically polarized, with intensity of 1, calculate the intensity after the light passes through a second polarizer sitting at an angle of 25^o to the vertical. How about 75^o to the vertical. HINT : To get the functional dependency, remember from class that a 45^o angle gives 0.5 of the original intensity.

3.

Which of the following equations represent an eigenfunction/eigenvalue situation? The operator is in parenthesis and the function follows it. For each example that is an eigenfunction/eigenvalue equation, give the eigenfunction; give the eigenvalue. The symbols "a" and "k" are constants. HINTS : Exp[x] is a way of phrasing e^x. If you don' t know how to multiply matrices, ask.

3.1. (x) Exp[x]

3.2. (d/dx) Exp[-a x]

3.3. (d/dθ) Sin[kθ]

3.4. $\left(d\hat{}2 \middle/ d\theta^2\right)$ Sin[kθ]

3.5. $\left(\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix} =$

3.6. $\left(\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix} =$

4.

In this exercise we apply two operators successively. The operators are again in parenthesis. You should (always) operate with the operator closest to the function first.

4.1. (y)(z) yz^2 =

4.2. (z)(y) yz^2 =

4.3. $(y)\left(\sqrt{\blacksquare}\right) yz^2$ =

4.4. $(\sqrt{\blacksquare})(y)\, yz^2$ =

4.5. When $O_1 O_2 f = O_2 O_1 f$, the operators are said to commute. When the equality does not hold, the operators do not commute. Do the operators y and z commute? Do the operators y and Sqrt[] commute?

5.

The following are some Mathematica exercises.

5.1. Enter the function $f = 3x^4 - 2x^2 + 4$ and evaluate it at x = 7.

5.2. Make a Table of values of f (from 5.1.) over the values of x from 0 to 10 in integer steps.

5.3. Find the derivative of f and evaluate it at x = 4.

5.4. Plot f over the range x = 0 to x = 3.

5.5. Integrate f from x = 0 to x = 5.

5.6. Do a numerical integration of f over the same limits as in 5.5.

6.

6.1. Find the de Broglie wavelength for a tennis ball (58 g) moving at 200 km/sec.

6.2. Find the de Broglie wavelength for an electron moving at a speed of 10^6 m/sec.

7.

7.1. You find a collection of old first class stamps in the office accumulated over the years from stamps left over when the Post Office raised postage. You have 4 $0.03 stamps, 8 $0.10, 3 $0.17, 3 $0.24, and 4 $0.29. What is the probability that you will, reaching randomly into this pile of stamps, pull out an $0.17 stamp?

7.2. What is the expectation value of the denomination of a stamp randomly withdrawn?

7.3. What is the expectation value of the square of the denomination of a stamp randomly withdrawn?

8.

8.1. Find the first 25 digits of π. HINT: M will do this with the command N[Pi, 25]. To get a list of those 25 digits you could do the command rr = RealDigits[N[Pi/10, 25]][[1]]. It might be useful for you to do these step at a time to see exactly what each does. HINT for HINT: the [[1]] takes the first element of a list. If you now use the command Count[rr, 3], you will count the occurrences of " 3"in the list; and if you are clever you can make a table that will list this information for all the digits between 0 and 9 all in one step.

8.2. What is the probability that you will get a 4 from a random pick of one of these 25 digits?

8.3. What is the most probable digit?

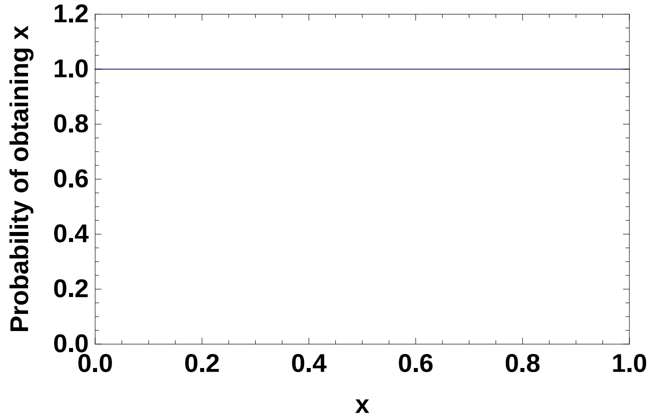
8.4. What is the average value of the digits (the expectation value)?

8.5. What is the standard deviation for the distribution in problem 8.1.?

9.

9.1. The plot below is the **probability** of obtaining a value of x, P(x), versus x. Use your intuition to determine the average value of x one would obtain with this probability function. HINT: You might pay attention to the commands used to do certain things in this plot.

```
Plot[1, {x, 0, 1}, PlotRange → {{0, 1}, {0, 1.2}},
 LabelStyle → {FontFamily → "Helvetica", Bold, FontSize → 12},
 Frame → True, FrameLabel → {"x", "Probability of obtaining x"}]
```

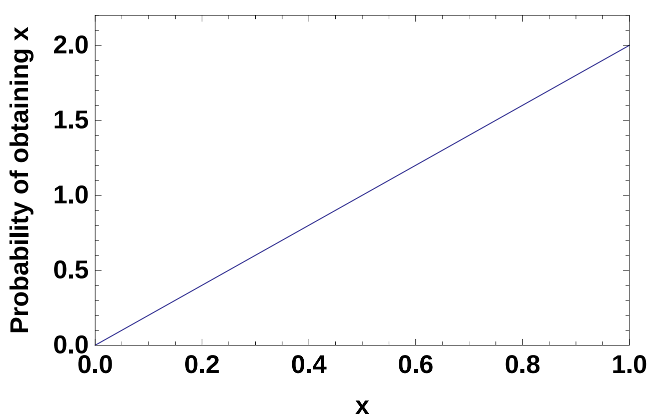


9.2. What is the probability you would find a value of x between 0 and 0.5?

10.

The plot below is of the relative probability of obtaining a value of x, P(x), versus x. Normalize the probability; determine the expectation value of x, $\langle x \rangle$, and the probability that one would find a value of x between 0 and 0.5?

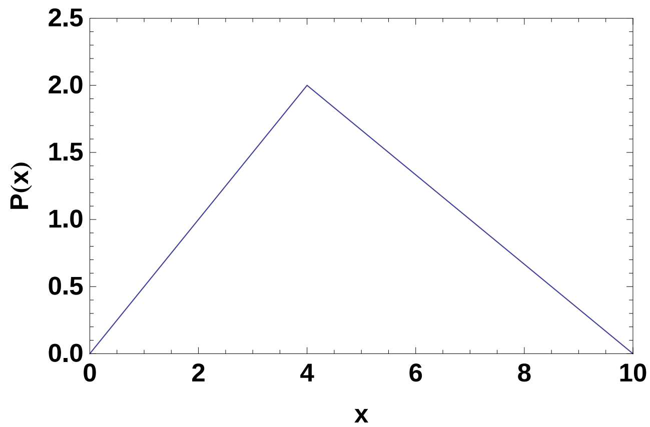
```
Plot[2 x, {x, 0, 1}, PlotRange → {{0, 1}, {0, 2.2}},
 LabelStyle → {FontFamily → "Helvetica", Bold, FontSize → 12},
 Frame → True, FrameLabel → {"x", "Probability of obtaining x"}]
```



11.

The following is a plot of the probability of obtaining a value of x, P(x), versus x. Normalize the probability; determine the expectation value of x, < x >, and the probability that one would find a value of x between 0 and 5? HINT : Because of the discontinuity at x = 4, you will have to divide your integral into two portions, 0 to 4 and 4 to 10.

```
yq[x_] := 0.5 x /; 0 ≤ x ≤ 4;
yq[x_] := 10 / 3 - 1 / 3 x /; 4 < x ≤ 10;
Plot[yq[x], {x, 0, 10}, PlotRange → {{0, 10}, {0, 2.5}},
 LabelStyle → {FontFamily → "Helvetica", Bold, FontSize → 12},
 Frame → True, FrameLabel → {"x", "P(x)"}]
```



12.

Let the probability function be

P(x) = A Exp[-α (x - q)^2]

Find the normalization parameter A, evaluate < x > and < x^2 >. HINT : the function extends to infinity in both directions.

13.

Imagine you are standing at the midpoint of the Boston marathon and you clock the velocity of the runners as they pass you. (Some of them are very tired by the time they get to the midpoint and are moving very slowly!) You find the probability of their velocity can be expressed as

P(v) = v Exp[-v/q]

where q is a positive constant and the direction of v is fixed and can range from 0 (because it is unlikely any runners are going to be going toward the starting line or at a diagonal to the route) to ∞ (for ease of integration).

13.1. Find the normalized probability expression for the velocity of the runners in terms of v and q.

13.2. What is the expectation value of the velocity, $< v >$?

13.3. What is the probability that a runner has a velocity of greater than four times the mean value of the velocity?

14.

A typical wave equation is

ψ(x, t) = A sin[2 π (x/λ - t/T]

where A, λ and T are constants.

14.1. Let A = 1, t = 0, λ = 2; plot the wave as a function of x from x = 0 to x = 6.

14.2. Plot the wave equation again for some later time, say t = 0.1 T.

14.3. Which way is the wave moving?

14.4. What would be the equation for a wave moving in the opposite direction?

15.

Two functions of x, f and g, are said to be orthogonal to each other over the range a to b if $\int_a^b f\, g\, dx = 0$.

15.1. For the functions f = sin[2 πx/λ] and g = sin[3 πx/λ], show they are orthogonal over the range 0 to 6 with λ = 6.

15.2. Using the functions from 15.1., plot each over the range 0 to 6.

15.3. Now plot the product of the two functions over the range 0 to 6 and examine the area under the curve, which is, afterall, the integral. Does the positive area cancel the negative?

15.4. Do the same for the functions sin[2 πx/λ] and cos[4 πx/λ] over the range of 0 to 8 with λ = 8.

16.

16.1. We have the following function:

$$y(x) = (8\, x^3 - 12\, x)\, \mathrm{Exp}\left[-x^2/2\right]$$

Plot this function between x = -5 and x = 5.

16.2. If you were to approximate the function in 16.1. by a Fourier series,

$$u(x) = a_0 + \sum_n (s_n\, \mathrm{Sin}[n\, \pi x/\lambda] + c_n\, \mathrm{Cos}[n\, \pi x/\lambda])$$

are there any terms that you would expect would go to zero? Why? HINT: This is a problem in logic, or, if you like, symmetry, not a M problem.

17.

17.1. Consider the function

$$y(x) = x(1 - x)^3$$

Plot this function between x = 0 and x = 1.

17.2. I claim that we can approximate this expression for y(x) by a sum of Sin terms :

$$y(x) \approx u(x) = \sum_{n=1}^{6} a_n \sqrt{2}\ \mathrm{Sin}[n\,\pi\,x]$$

where the a_n are numbers. You showed in exercise 15 that Sin terms with different n values are orthogonal to each other. What would happen, therefore, if you evaluated the integral $\int_0^1 u(x) \sqrt{2}\ \mathrm{Sin}[2\,\pi\,x]\, dx$?

17.3. If the approximation is reasonable, what would you get if you took the integral $\int_0^1 y(x) \sqrt{2}\ Sin[2\,\pi\ x]\, dx$?

17.4. Hopefully you learned in the exercise 17.3. how to evaluate a_n values. Find the six a_n values. HINT: This cries out for a M. table.

17.5. Plot y(x) and your approximation for it on the same plot to see how good the approximation is.

18.

A time dependent wave function can be written:

ψ(x, t) =
Sqrt[2/L]{0.9 Sin[π x/L]Exp[-i w t] + 0.2Sin[3 π x/L]Exp[-i 9 w t]

Let w = 3.22 sec^{-1}and L = 6. Evaluate the time dependency of the probability, which we will define shortly as ψ(x, t)*ψ(x, t), where the “ star”is the complex conjugate. Plot the probability (as a function of x between 0 and L) for t = 0, 0.1, 0.2, 0.3, 0.4, and 0.5 sec to verify that the probability is a function of time. At what time, if any, does the answer become the same as at t = 0?

19.

19.1. A standing wave has the equation

$$\psi\,(x, t) = A \sin[2\,\pi\, x/\lambda] \cos[2\,\pi\, t/T]$$

For A = 1, l = 2, t = 0, plot this function from x = 0 to x = 6. Do the same for t = 0.1 T, 0.24 T, and 0.29 T. Is it a standing wave?

19.2. Is the wave A sin[kx - wt] + 2 Asin[kx + wt] stationary? Show by expansion of the sin function. Then verify it numerically (with Mathematica) using k = 2, w = 1, and t = 0 and 1.