Exponential Growth in Ebola Outbreak Since May 14, 2014

C ontrary to general opinion, the current Ebola outbreak in West Africa followed an exponential growth curve starting already in mid-May. The death toll followed an exponential growth curve with almost the same time constant, allowing direct calculation of the mortality of the outbreak. This value remained steady at about 72%, contrary to the estimate of the World Health Organization of slightly above 50%. Until the last 2 weeks, the projected date at which the number of infected individuals would reach 100,000 had remained steady at January 19. Updated statistics from September 6 advanced that date by at least a month. Estimates suggest that over 20,000 already have been infected, exceeding the number that the WHO has declared could be the eventual outcome.

The statistics of the Ebola outbreak in western Africa, as reported by the World Health Organization (WHO), has gone through three phases, as shown in Figure 1, and may be entering a fourth. The first several months saw a relatively rapid increase, followed by a period of about 6 weeks when the disease appeared to be under control. But from the end of May to the end of August, the outbreak followed an almost ideal exponential growth rate. With such unambiguous statistics, several conclusions could be drawn that need rapid transmission. First, however, some basic results from previous Ebola epidemics are introduced in order to provide a context for discussion.

Although the incubation period of Ebola is known to range from 2 to 21 days [1, 2], the typical incubation period is 8–10 days. When fatal, death usually occurs between 7 and 16 days, and most often between 8 and 9 days [3] after the onset of symptoms. When not fatal, the acute phase typically lasts 14–16 days [3]. Thus, in cases where death is the outcome, the typical course of the disease is approximately 17–18 days, and when the patient recovers, about 24 days. The disease can be transmitted at any time when symptoms are apparent. Further, fluids from the human body remain contagious some additional days past death, allowing infection of other individuals posthumously, which does occur for a variety of reasons, including respectful handling of the body. A typical period between diagnosis of a primary case and a secondary case is thus a mean incubation period (10 days) plus some fraction, perhaps half of the period the individual remains infectious (roughly an additional 7 days). The finite course of the disease means that the cumulative deaths at any given time, generates the mortality statistics for the cases 18.5 days earlier (approximately 10 days incubation plus 8.5 days until death though the number of deaths at a given data relates to the number of reported cases about 8.5 days earlier). Either of these estimates carries the uncertainty associated with skewed distributions of transmission times or incubation periods [4], as well as an uncertainty related to the actual values of the time intervals.

ALLEN G. HUNT

Allen G. Hunt is at Department of Physics, Wright State University, Dayton, Ohio (E-mail: allen.hunt@wright.edu)

FIGURE 1

West African Ebola outbreak, 2014.
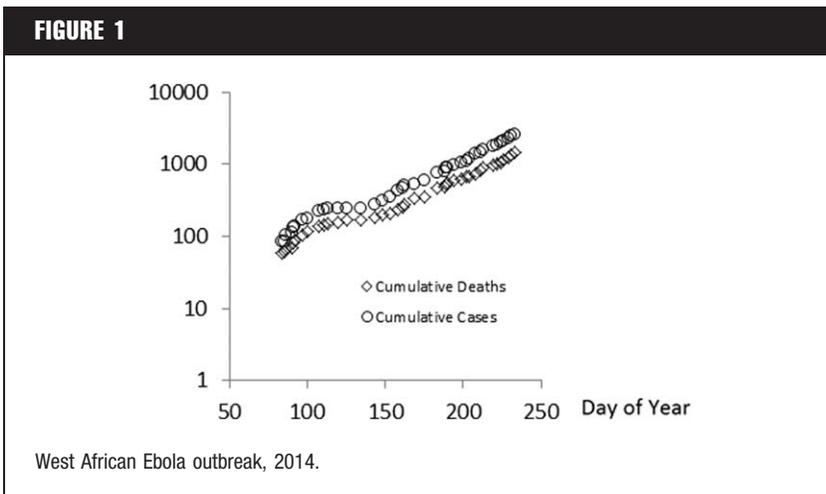
In particular, Breman et al. [5], give a shorter incubation period, and Birmingham et al. [6], cite a shorter interval between infection and death. Subsequent calculations here, however, use the numbers given above, as they are currently cited by the Center for Disease Control and the World Health Organization, and because uncertainty in these values does not change statistical predictions of the number of cases and deaths. But some of the detailed results given in the following interpretation would be changed if different time intervals were used.
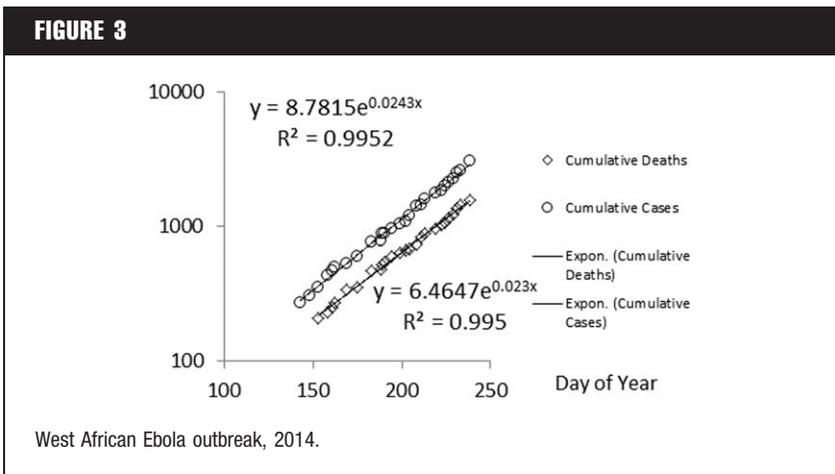
The duration of the infectious period is relevant for understanding the number, R_0, of secondary cases an index case can generate. R_0, also called the reproductive number, has taken on values less than 2 in previous outbreaks (e.g., [4]), but exponential growth is generated for any $R_0 > 1$. Control is achieved if R_0 is maintained at a value less than 1, after which an epidemic will eventually end. In the past, Ebola outbreaks have been swiftly controlled, and the initial exponential growth phase was quickly overcome. Outbreak epidemiologists (e.g., [4]) have developed Markov chain models in conjunction with differential equations, with which they explain the exponential growth. Such models typically have means to account for the reduction in R_0 that accompanies
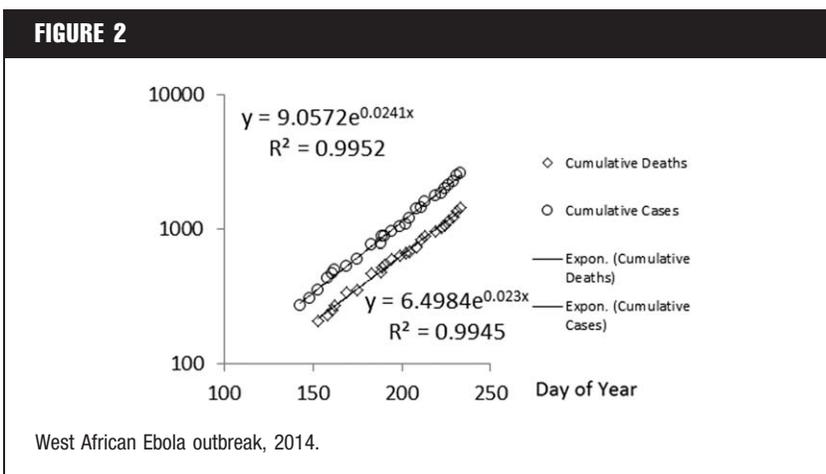
intervention, including the ability to control an epidemic when the reproductive number drops below 1. In the current outbreak, however, the evidence is that R_0 has increased recently, after maintaining a constant value near 1.5 through 3 months of inadequate intervention.

Consider that by the beginning of June, Doctors Without Borders had concluded that the outbreak was out of control. Figure 2 shows, starting at day (of the year) 134, the logarithm of the total number of cases (confirmed plus suspected), as well as, starting at day (of the year) 153, the logarithm of the number of deaths attributed to Ebola, both as functions of time. The source of these statistics is also the WHO web pages for the West Africa Ebola Outbreak. Note that the general conclusions below do not depend sensitively on the choice of the starting date, as long as it is chosen beyond the interval where the outbreak appeared to be contained. Both growth curves follow nearly the same exponential function, with time constants 0.0243/day and 0.023/day, respectively. The prefactors are 9.06 and 6.50, respectively. The ratio of fatality to case prefactors, 0.717, is a good first estimate of the mortality of the current Ebola outbreak. The choice of a delay of 19 days in the initial day for the death curve is almost identical to the 18.5 day delay between infection and mortality. Both time constants are consistent with a doubling period of about 29 days. If the doubling period of the disease were the mean duration of the disease, one could make a rough estimate that each patient infected two others. However, the doubling period was, and is still, longer, meaning that $R_0 < 2$. In such a case a first estimate of the reproductive number involves the generational delay. If the period 17 days is used, then $R_0 \approx \exp[(0.0243)(17)] = 1.51$ results. This value is intermediate between the values 1.83 in an outbreak in the Congo, and 1.34 in an outbreak in Uganda [4]. The date of

FIGURE 2

West African Ebola outbreak, 2014.

FIGURE 3

$$y = 8.7815e^{0.0243x}$$
$$R^2 = 0.9952$$

◇ Cumulative Deaths
○ Cumulative Cases
— Expon. (Cumulative Deaths)
— Expon. (Cumulative Cases)

$$y = 6.4647e^{0.023x}$$
$$R^2 = 0.995$$

Day of Year

West African Ebola outbreak, 2014.

the analysis was August 22, 2014. In the initial phase of the epidemic, before any intervention, however, the value of R_0 calculated by the same means was 2.26.

The most important implication of Figure 2 (knowledge available to the public by August 22) was that, had that trend continued, the total number of cases would have reached 100,000 by day 384, January 19, 2015. Twenty days later, on February 8, the total number of deaths would have reached 72,000. The fact that a delay of 20 days is required for the fatalities to reach 72% of the total number of cases is in general accord with the typical delay between

infection and death, about 18.5 days. More importantly, by early next year, at the current requirement of tracking up to 20 or more contacts per patient, it would become necessary to track closely the health of up to two million people. Clearly, an emergency response far beyond what is presently in place is necessary to avoid such a scenario.

Five days later (Figure 3) on August 27, the total deaths had risen to 1552, and the total cases to 3069. Repetition of the analysis above merely changed the mortality rate to 73.6%, but the date for which the number of cases reaches 100,000 remained the same, January 19, whereas the date at which
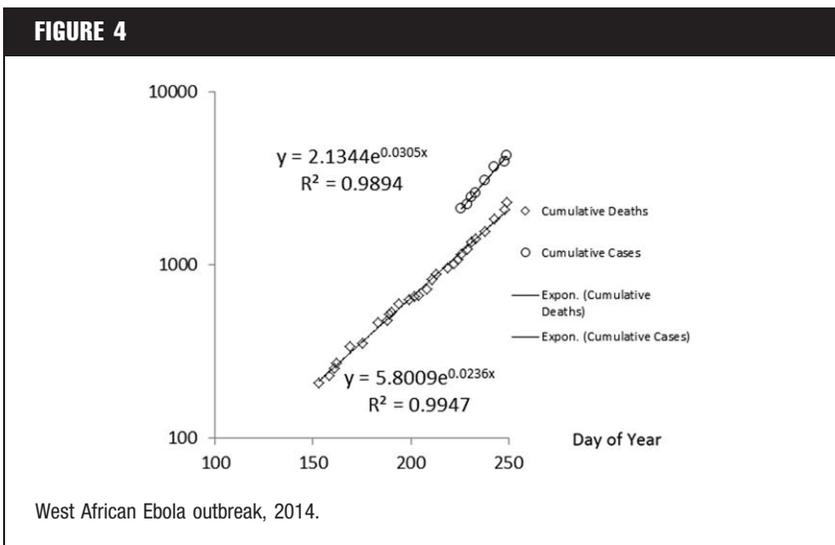
73.6% of those cases would have been fatal changed one day to February 9. If, instead, the mortality rate of the initial analysis, 71.7%, is chosen, the date at which 72,000 deaths is reached is also unchanged at February 8. The constancy of the projections indicated the robustness of the estimates and suggested that increases in intervention were not sufficient to reduce the reproduction number.

If the current WHO estimates that only $^1/_4$ to $^1/_2$ of the cases and deaths are actually detected is correct, the analysis changes only in the dates at which these catastrophic measures are reached. If the ratio is $^1/_2$, then the total cases would reach 100,000 29 days earlier on December 21; if the ratio is $^1/_4$, this figure would be reached on November 22.

Updated caseload statistics through September 6, 2014, however, suggest a further deterioration in the situation. Although taken together with the earlier statistics, they could appear to fit the same exponential function, taken separately they imply a more rapid increase (Figure 4). Here, the time for case doubling diminished from 29 days to 20 days, commensurate with an increase in R_0 from 1.51 to 1.68, and the total caseload of 100,000 would be reached already by December 16. That such an increase in transmission actually occurred, is suggested by the fact that in the first 20 days of this increase in the case load, the exponential function for the death rate did not change. Including the potential underestimation of detected cases by the factor $^1/_4$ advances the date for 100,000 cases 40 days to November 6, and a total of 1.6 million cases would be reached by January 25. Note that it is not a high reproductive number which is causing the difficulty in containment, but the large number of cases, indicating that an early response was critical.

It will be important to determine what the cause of the recent spike in

FIGURE 4

$$y = 2.1344e^{0.0305x}$$
$$R^2 = 0.9894$$

◇ Cumulative Deaths
○ Cumulative Cases
— Expon. (Cumulative Deaths)
— Expon. (Cumulative Cases)

$$y = 5.8009e^{0.0236x}$$
$$R^2 = 0.9947$$

Day of Year

West African Ebola outbreak, 2014.

cases is. One possibility is that a portion of the "hidden" caseload discussed by the WHO has become more visible. If the increase reflects the above change in reproductive ratio, however, the inferences are progressively worse. Only about 185 days into the New Year, the number of cases would reach 100 million, if the current growth rate was allowed to continue, and this number would be reached 40 days earlier in the case of the feared underreporting of cases.

We must consider the implications of these statistics. First, if the number of cases is underestimated by $\frac{1}{4}$, as the WHO has suggested possible, the most recent reported number, 4293, of cases on September 6, implies that already close to 17,000 would be infected. Added to that the number of cases in the incubation stage (and the fact that another 4 days have passed), the cumulative number of cases would almost certainly already exceed 20,000, the value that the WHO has suggested could represent the peak of the outbreak. Second, using a rough estimate of the necessity of tracking 20 people per infected person, the number of people that need to be tracked is already close to a half a million. The resources needed to track this number of people, and twice as many within 20 days, is likely almost beyond the capability of response. Alternative means to control such an epidemic should be avoided if at all possible, however.

It should be emphasized in conclusion that the statistical analyses reported here were performed on the total number of cases for all countries affected in the West Africa outbreak. The World Health Organization reported statistics for each affected region separately. Separate analyses of the individual regions could have obscured the consistency of the general analysis. Nevertheless, if progress is currently being made in containment in any subset of the affected countries, in order to guide the international response, such individual analyses will also need to be performed in distinct regions.

Finally, the reason why most epidemiological responses have included the possibility that the reproductive number can be decreased to below 1 is that, in the past, each outbreak of Ebola has been brought under control. International agencies are bringing the same techniques to bear on this epidemic. These include contact tracing, patient isolation, and possibly even new medicines. Other social responses of the affected public can reduce the spread of infected individuals as well. Thus, there is no certainty that the present exponential trends will continue and, indeed, hope that it will not.

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